Online appendix for "Patent protection, innovation, and technology transfer in a Schumpeterian economy"

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Appendix A : Proof of propositions and lemmas

Appendix B : An alternative specification of R&D

Appendix C : Transitional dynamics

This appendix charaterizes the dynamical system of the baseline model. Define four transformed variables $\{z_{1,t}, z_{2,t}, z_{3,t}, z_{4,t}\}$ by

$$z_{1,t} \equiv \frac{L_t}{Q_t^{1-\xi}}; \quad z_{2,t} \equiv \frac{A_t^N}{Y_t}; \quad z_{3,t} \equiv \frac{A_t^S}{Y_t}; \quad z_{4,t} \equiv \frac{Q_t^N}{Q_t^S},$$

where $A_t^N \equiv a_t^N L_t^N$ and $A_t^S \equiv a_t^S L_t^S$ are the total asset value of households in the North and the South, respectively. Thus, $Q_t^N/Q_t = Q_t^N/(Q_t^N + Q_t^S) = z_{4,t}/(1 + z_{4,t})$ and $Q_t^S/Q_t = 1/(1 + z_{4,t})$.

Taking the log of $z_{1,t}$ and differentiating the resulting equation with respect to time yields

$$\frac{\dot{z}_{1,t}}{z_{1,t}} = g_L - (1-\xi)\frac{\dot{Q}_t}{Q_t} = g_L - (1-\xi)(\kappa-1)\lambda_t^N = g_L - (1-\xi)(\kappa-1)\frac{1-\alpha}{\beta} z_{1,t} l_{r,t}^N, \tag{C.1}$$

where

$$\lambda_t^N = \frac{L_{r,t}^N}{\beta Q_t^{1-\xi}} = \frac{1}{\beta}\frac{L_{r,t}^N}{L_t^N}\frac{L_t^N}{L_t}\frac{L_t}{Q_t^{1-\xi}} = \frac{1-\alpha}{\beta} z_{1,t} l_{r,t}^N \tag{C.2}$$

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is derived from (33) and $l_{r,t}^N \equiv L_{r,t}^N / L_t^N$ is the share of R&D labor in the North.

Similarly, differentiating the log of $z_{2,t}$ and $z_{3,t}$ with respect to time yields

$$\frac{\dot{z}_{2,t}}{z_{2,t}} = \frac{\dot{A}_t^N}{A_t^N} - \frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t^N}{A_t^N} - (r_t - \rho) - g_L \quad \text{and} \quad \frac{\dot{z}_{3,t}}{z_{3,t}} = \frac{\dot{A}_t^S}{A_t^S} - (r_t - \rho) - g_L, \tag{C.3}$$

where the condition $\dot{c}_t^N / c_t^N = \dot{c}_t^S / c_t^S = \dot{Y}_t / Y_t - \dot{L}_t / L_t$ implied by (5) and the Euler equation (3) have been applied. Moreover, from (2), we can show that $\dot{A}_t^N = r_t A_t^N + w_t^N L_t^N - c_t^N L_t^N$ and $\dot{A}_t^S = r_t A_t^S + w_t^S L_t^S - c_t^S L_t^S$, which implies that

$$\frac{\dot{A}_t^N}{A_t^N} = r_t + \frac{w_t^N L_t^N}{A_t^N} - \frac{c_t^N L_t^N}{A_t^N} \quad \text{and} \quad \frac{\dot{A}_t^S}{A_t^S} = r_t + \frac{w_t^S L_t^S}{A_t^S} - \frac{c_t^S L_t^S}{A_t^S}, \tag{C.4}$$

where $A_t^N = \beta w_t^N Q_t^{1-\xi}$ and $A_t^S = \gamma w_t^S Q_t^S Q_t^{-\xi}$ are given by (A.4) and (A.5), respectively. To derive $c_t^N L_t^N / A_t^N$, we first combine the aggregate form of (21) with $\dot{A}_t^N = r_t A_t^N + w_t^N L_t^N - c_t^N L_t^N$ yielding

$$r_t A_t^N + w_t^N L_t^N - c_t^N L_t^N = r_t v_t^N - \pi_t^N + w_t^N L_{r,t}^N \Leftrightarrow c_t^N L_t^N = w_t^N L_t^N + \pi_t^N - \lambda_t^N v_t^N. \tag{C.5}$$

Substituting (C.5) back into (C.4), together with the definition of A_t^N, gives rise to

$$\begin{aligned} \frac{\dot{A}_t^N}{A_t^N} &= r_t + \lambda_t^N - \frac{\pi_t^N}{A_t^N} \\ &= r_t + \lambda_t^N - \frac{(\mu^N - 1)(\mu^N)^{-\sigma} Q_t (w_t^N)^{1-\sigma} Y_t}{\beta w_t^N Q_t^{1-\xi}} \\ &= r_t + \lambda_t^N - \frac{(\mu^N - 1) L_{x,t}^N}{\beta Q_t^{1-\xi}} \frac{Q_t}{Q_t^N} \\ &= r_t + \frac{1-\alpha}{\beta} z_{1,t} l_{r,t}^N - \frac{(1-\alpha)(\mu^N - 1)(1 + z_{4,t})}{\beta z_{4,t}} l_{x,t}^N z_{1,t} \\ &= r_t + \frac{1-\alpha}{\beta} z_{1,t} \left[1 - \mu^N l_{x,t}^N - \frac{1}{z_{4,t}} (\mu^N - 1) l_{x,t}^N \right], \end{aligned} \tag{C.6}$$

where we have applied (15) in the second equality, (12) and (14) to substitute for the expression of $L_{x,t}^N$ in the third equality, (C.2) in the forth equality, and $l_{x,t}^N \equiv L_{x,t}^N / L_t^N = 1 - l_{r,t}^N$ in the last equality. Then, we substitute (C.6) into (C.3) to rewrite $\dot{z}_{2,t} / z_{2,t}$ as

$$\frac{\dot{z}_{2,t}}{z_{2,t}} = \rho - g_L + \frac{1-\alpha}{\beta} z_{1,t} \left[1 - \mu^N l_{x,t}^N - \frac{1}{z_{4,t}} (\mu^N - 1) l_{x,t}^N \right]. \tag{C.7}$$

Similarly, to derive $c_t^S L_t^S / A_t^S$, we first aggregate (22) such that $\int_{\theta_t^S} \dot{v}_t^S(j) dj = r_t \int_{\theta_t^S} v_t^S(j) dj -$

$\int_{\theta_t^S} \pi_t^S(j)dj + \lambda_t^N \int_{\theta_t^S} v_t^S(j)dj$. Using this equation, we then have

$$\begin{aligned}
\dot{A}_t^S &= \int_{\theta_t^S} [\dot{v}_t^S(j) - \dot{v}_t^N(j)]dj = (r_t + \lambda_t^N) \int_{\theta_t^S} v_t^S(j)dj - \int_{\theta_t^S} \pi_t^S(j)dj - \left[(r_t + \lambda^N) \int_{\theta_t^S} v_t^N(j)dj - \int_{\theta_t^S} \pi_t^N(j)dj \right] \\
&= (r_t + \lambda_t^N) \int_{\theta_t^S} [v_t^S(j) - v_t^N(j)]dj - \int_{\theta_t^S} [\pi_t^S(j) - \pi_t^N(j)]dj \\
&= (r_t + \lambda_t^N) \int_{\theta_t^S} \gamma w_t^S q_t(j) Q_t^{-\xi} dj - \int_{\theta_t^S} (\mu^S - 1) w_t^S q_t(j) \frac{\overline{L}_{x,t}^S}{Q_t} dj + \int_{\theta_t^S} (\mu^N - 1) w_t^N q_t(j) \frac{\overline{L}_{x,t}^N}{Q_t} dj \\
&= (r_t + \lambda_t^N) \gamma w_t^S Q_t^S Q_t^{-\xi} - (\mu^S - 1) w_t^S L_{x,t}^S + (\mu^N - 1) w_t^N L_{x,t}^N \frac{Q_t^N}{Q_t},
\end{aligned} \tag{C.8}$$

where we have used (15) and (16) in the third equality. By combining (C.8) and $\dot{A}_t^S = r_t A_t^S + w_t^S L_t^S - c_t^S L_t^S$, we obtain

$$\begin{aligned}
r_t A_t^S + w_t^S L_t^S - c_t^S L_t^S &= (r_t + \lambda_t^N) A_t^S - \left[(\mu^S - 1) w_t^S L_{x,t}^S - (\mu^N - 1) w_t^N L_{x,t}^N \frac{Q_t^S}{Q_t^N} \right] \\
\Leftrightarrow c_t^S L_t^S &= w_t^S L_t^S - \lambda_t^N A_t^S + \left[(\mu^S - 1) w_t^S L_{x,t}^S - (\mu^N - 1) w_t^N L_{x,t}^N \frac{Q_t^S}{Q_t^N} \right].
\end{aligned} \tag{C.9}$$

Inserting (C.9) back into (C.4) yields

$$\begin{aligned}
\frac{\dot{A}_t^S}{A_t^S} &= r_t + \lambda_t^N - \frac{(\mu^S - 1) w_t^S L_{x,t}^S - (\mu^N - 1) w_t^N L_{x,t}^N \frac{Q_t^S}{Q_t^N}}{\gamma w_t^S Q_t^S Q_t^{-\xi}} \\
&= r_t + \lambda_t^N - \frac{(\mu^S - 1) \frac{L_{x,t}^S}{L_t^S} \frac{L_t^S}{L_t} - (\mu^N - 1) \omega_t \frac{L_{x,t}^N}{L_t^N} \frac{L_t^N}{L_t} \frac{1}{z_{4,t}}}{\gamma \frac{Q_t^S}{Q_t} \frac{Q_t^{1-\xi}}{L_t}} \\
&= r_t + \frac{1-\alpha}{\beta} z_{1,t} l_{r,t}^N - \frac{\alpha(\mu^S - 1) l_{x,t}^S - (\mu^N - 1)(1-\alpha) \omega_t l_{x,t}^N \frac{1}{z_{4,t}}}{\gamma \frac{1}{1+z_{4,t}} \frac{1}{z_{1,t}}} \\
&= r_t + \frac{1-\alpha}{\beta} z_{1,t} l_{r,t}^N - \frac{\alpha(\mu^S - 1) l_{x,t}^S (1 + z_{4,t}) - (\mu^N - 1)(1-\alpha) \omega_t l_{x,t}^N \frac{1+z_{4,t}}{z_{4,t}}}{\gamma} z_{1,t},
\end{aligned} \tag{C.10}$$

where we have used $l_{x,t}^S \equiv L_{x,t}^S / L_t^S = 1 - l_{r,t}^S$ and (16) in the second equality, (13) and (14) to substitute for the expression of $L_{x,t}^S$ in the third equality, and (C.2) in the last equality. By applying (C.10), we can rewrite $\dot{z}_{3,t}/z_{3,t}$ as in (C.3) as

$$\frac{\dot{z}_{3,t}}{z_{3,t}} = \rho - g_L + \frac{1-\alpha}{\beta} z_{1,t} l_{r,t}^N - \frac{\alpha(\mu^S - 1) l_{x,t}^S (1 + z_{4,t}) - (\mu^N - 1)(1-\alpha) \omega_t l_{x,t}^N \left(\frac{1+z_{4,t}}{z_{4,t}} \right)}{\gamma} z_{1,t}. \tag{C.11}$$

Then taking the log of $z_{4,t}$ and differentiating the resulting equation with respect to time yield

$$\frac{\dot{z}_{4,t}}{z_{4,t}} = \frac{\dot{Q}_t^N}{Q_t^N} - \frac{\dot{Q}_t^S}{Q_t^S} = (\kappa - 1)\lambda_t^N + \kappa\lambda_t^N \frac{Q_t^S}{Q_t^N} - \lambda_t^S - \lambda_t^S \frac{Q_t^N}{Q_t^S} + \lambda_t^N = \lambda_t^N \left(\kappa - 1 + \frac{\kappa}{z_{4,t}}\right) - \lambda_t^S(1 + z_{4,t}), \tag{C.12}$$

where (A.1) and (A.2) have been used in the second equality, and according to (37), λ_t^S is a function of $l_{r,t}^S$, $z_{1,t}$, and $z_{4,t}$ such that

$$\lambda_t^S = \frac{L_{r,t}^S}{\gamma Q_t^{1-\xi}} \frac{Q_t}{Q_t^N} = \frac{1}{\gamma} \frac{L_{r,t}^S}{L_t^S} \frac{L_t^S}{L_t} \frac{L_t}{Q_t^{1-\xi}} \frac{Q_t}{Q_t^N} = \left(\frac{\alpha}{\gamma}\right) l_{r,t}^S z_{1,t} \left(1 + \frac{1}{z_{4,t}}\right). \tag{C.13}$$

Notice that the differential equations (C.1), (C.7), (C.11) and (C.12) contain seven endogenous variables in total, namely $\{z_{1,t}, z_{2,t}, z_{3,t}, z_{4,t}, l_{x,t}^N, l_{x,t}^S, \omega_t\}$. However, it will be shown that, in the following steps, variables $\{l_{x,t}^N, l_{x,t}^S, \omega_t\}$ are implicit functions of $\{z_{1,t}, z_{2,t}, z_{3,t}, z_{4,t}\}$. Hence, (C.1), (C.7), (C.11) and (C.12) constitute a system of autonomous differential equations characterizing the model's dynamical behaviors.

First, we express the relative wage rate ω_t as a function of $z_{3,t}$, $l_{x,t}^N$, and $l_{x,t}^S$. From (12), (13), and (14), we have

$$\frac{l_{x,t}^N}{l_{x,t}^S} = \frac{\frac{Q_t^N \overline{L}_{x,t}^N}{Q_t L_t^N}}{\frac{Q_t^S \overline{L}_{x,t}^S}{Q_t L_t^S}} = \frac{\alpha}{1-\alpha} \frac{\overline{L}_{x,t}^N}{\overline{L}_{x,t}^S} z_{4,t} = \frac{\alpha}{1-\alpha} \delta^{1-\sigma} \frac{(\mu^N w_t^N)^{-\sigma}}{(\mu^S w_t^S)^{-\sigma}} z_{4,t} = \frac{\alpha}{1-\alpha} \delta^{1-\sigma} \left(\frac{\mu^S}{\mu^N}\right)^{\sigma} \omega_t^{-\sigma} z_{4,t},$$

and thus

$$\omega_t = \left(\frac{l_{x,t}^S}{l_{x,t}^N}\right)^{1/\sigma} \left(\frac{\alpha}{1-\alpha}\right)^{1/\sigma} \delta^{\frac{1-\sigma}{\sigma}} \left(\frac{\mu^S}{\mu^N}\right) z_{4,t}^{1/\sigma}. \tag{C.14}$$

Next, we construct two equations of $l_{x,t}^N$ and $l_{x,t}^S$ that are solved by $\{z_{1,t}, z_{2,t}, z_{3,t}, z_{4,t}\}$. We substitute (7) and (8) into (4) to reexpress the equilibrium final-good production function as

$$
\begin{aligned}
Y_t &= \left\{ \int_{\theta_t^N} [x_t^N(j)]^{\frac{\sigma-1}{\sigma}} dj + \int_{\theta_t^S} [x_t^S(j)]^{\frac{\sigma-1}{\sigma}} dj \right\}^{\frac{\sigma}{\sigma-1}} = \left\{ \int_{\theta_t^N} [z^{n_t(j)} L_{x,t}^N(j)]^{\frac{\sigma-1}{\sigma}} dj + \int_{\theta_t^S} [z^{n_t(j)} \delta L_{x,t}^S(j)]^{\frac{\sigma-1}{\sigma}} dj \right\}^{\frac{\sigma}{\sigma-1}} \\
&= \left\{ \int_{\theta_t^N} [q_t(j)]^{1/\sigma} [q_t(j) \overline{L}_{x,t}^N / Q_t]^{\frac{\sigma-1}{\sigma}} dj + \int_{\theta_t^S} [q_t(j)]^{1/\sigma} [\delta q_t(j) \overline{L}_{x,t}^S / Q_t]^{\frac{\sigma-1}{\sigma}} dj \right\}^{\frac{\sigma}{\sigma-1}} \\
&= \left\{ \int_{\theta_t^N} q_t(j) Q_t^{\frac{1-\sigma}{\sigma}} (\overline{L}_{x,t}^N)^{\frac{\sigma-1}{\sigma}} dj + \int_{\theta_t^S} q_t(j) Q_t^{\frac{1-\sigma}{\sigma}} (\delta \overline{L}_{x,t}^S)^{\frac{\sigma-1}{\sigma}} dj \right\}^{\frac{\sigma}{\sigma-1}} = \left\{ (\overline{L}_{x,t}^N)^{\frac{\sigma-1}{\sigma}} Q_t^N + (\delta \overline{L}_{x,t}^S)^{\frac{\sigma-1}{\sigma}} Q_t^S \right\}^{\frac{\sigma}{\sigma-1}} \frac{1}{Q_t} \\
&= \left\{ \left(\frac{\overline{L}_{x,t}^N Q_t^N}{Q_t L_t^N} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Q_t^N}{Q_t L_t^N} \right)^{\frac{1-\sigma}{\sigma}} Q_t^N + \delta^{\frac{\sigma-1}{\sigma}} \left(\frac{\overline{L}_{x,t}^S Q_t^S}{Q_t L_t^S} \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Q_t^S}{Q_t L_t^S} \right)^{\frac{1-\sigma}{\sigma}} Q_t^S \right\}^{\frac{\sigma}{\sigma-1}} \frac{1}{Q_t} \\
&= \underbrace{\left\{ (l_{x,t}^N)^{\frac{\sigma-1}{\sigma}} (1-\alpha)^{\frac{\sigma-1}{\sigma}} \left(\frac{z_{4,t}}{1+z_{4,t}} \right)^{1/\sigma} + (\delta\alpha)^{\frac{\sigma-1}{\sigma}} (l_{x,t}^S)^{\frac{\sigma-1}{\sigma}} \left(\frac{1}{1+z_{4,t}} \right)^{1/\sigma} \right\}^{\frac{\sigma}{\sigma-1}}}_{\Xi} \left(Q_t^{\frac{1}{\sigma-1}} L_t \right)
\end{aligned}
\tag{C.15}
$$

where (14), (32), and (36) have been applied in sequence. From (15), the aggregate expenditure on Northern production labor is given by

$$
w_t^N L_{x,t}^N = \frac{w_t^N Q_t^N Y_t}{(\mu^N w_t^N)^\sigma} \Leftrightarrow w_t^N = \frac{1}{\mu^N} \cdot \left(\frac{Q_t^N Y_t}{L_{x,t}^N} \right)^{1/\sigma}. \tag{C.16}
$$

From (18), we can derive the total value of innovative R&D firms such that

$$
\lambda_t^N = \frac{w_t^N L_{r,t}^N}{v_t^N} \Leftrightarrow v_t^N = \frac{L_{r,t}^N}{\lambda_t^N \mu^N} \left(\frac{Q_t^N Y_t}{L_{x,t}^N} \right)^{1/\sigma}, \tag{C.17}
$$

where the last equality is obtained by using (C.16). Thus, we have

$$
\begin{aligned}
z_{2,t} = \frac{v_t^N}{Y_t} &= \frac{L_{r,t}^N}{\lambda_t^N \mu^N} \left(\frac{Q_t^N}{L_{x,t}^N} \right)^{1/\sigma} Y_t^{\frac{1-\sigma}{\sigma}} \\
&= \frac{l_{r,t}^N (1-\alpha) L_t}{\mu^N \lambda_t^N} \left(\frac{Q_t^N}{Q_t} \right)^{1/\sigma} \cdot Q^{1/\sigma} \left(\frac{1}{l_{x,t}^N (1-\alpha) L_t} \right)^{1/\sigma} \Xi^{\frac{1-\sigma}{\sigma}} Q_t^{-1/\sigma} L_t^{\frac{1-\sigma}{\sigma}} \\
&= \frac{\beta}{\mu^N z_{1,t}} \left(\frac{z_{4,t}}{1+z_{4,t}} \right)^{1/\sigma} \Xi^{\frac{1-\sigma}{\sigma}} (1-\alpha)^{-1/\sigma} \left(l_{x,t}^N \right)^{-1/\sigma}.
\end{aligned}
\tag{C.18}
$$

where (C.2) and (C.15) have been applied. (C.18) is then the first equation that solves for $l_{x,t}^N$ and $l_{x,t}^S$. Similarly, from (16), the aggregate expenditure on Southern production labor is given by

$$
w_t^S L_{x,t}^S = \delta^{\sigma-1} \frac{w_t^S Q_t^S Y_t}{(\mu^S w_t^S)^\sigma} \Leftrightarrow w_t^S = \frac{\delta^{\frac{\sigma-1}{\sigma}}}{\mu^S} \left(\frac{Q_t^S Y_t}{L_{x,t}^S} \right)^{1/\sigma}. \tag{C.19}
$$

The total value of adaptive R&D firms is derived by using (20) such that

$$\lambda_t^S \cdot \int_{\theta_t^N} [v_t^S(j) - v_t^N(j)]dj = w_t^S \int_{\theta_t^N} L_{r,t}^S(j)dj \Leftrightarrow \lambda_t^S \left(\frac{\theta_t^N}{\theta_t^S}\right) A_t^S = w_t^S L_{r,t}^S$$
$$\Leftrightarrow A_t^S = \frac{w_t^S L_{r,t}^S}{\lambda_t^N} = \frac{L_{r,t}^S}{\lambda_t^N \mu^S} \delta^{\frac{\sigma-1}{\sigma}} \left(\frac{Q_t^S Y_t}{L_{x,t}^S}\right)^{1/\sigma}, \tag{C.20}$$

where the relation $\theta_t^N/\theta_t^S = \lambda_t^N/\lambda_t^S$ implied by (25), (26), and (C.19) has been used in sequence. Therefore, using (C.15) and (C.20) allows us to rewrite $z_{3,t}$ as

$$z_{3,t} = \frac{A_t^S}{Y_t} = \frac{L_{r,t}^S}{\lambda_t^N \mu^S} \delta^{\frac{\sigma-1}{\sigma}} \left(\frac{Q_t^S}{L_{x,t}^S}\right)^{1/\sigma} Y_t^{\frac{1-\sigma}{\sigma}}$$
$$= \frac{\beta(1-l_{x,t}^S)}{\mu^S z_{1,t}(1-\alpha)(1-l_{x,t}^N)} \left(\frac{1}{1+z_{4,t}}\right)^{\frac{1}{\sigma}} \Xi^{\frac{1-\sigma}{\sigma}} \delta^{\frac{\sigma-1}{\sigma}} \alpha^{\frac{\sigma-1}{\sigma}} \left(l_{x,t}^S\right)^{-\frac{1}{\sigma}} \tag{C.21}$$

(C.21) is then the second equation that solves for $l_{x,t}^N$ and $l_{x,t}^S$. Thus, (C.18) and (C.21) imply that both $l_{x,t}^N$ and $l_{x,t}^S$ are implicit functions of $z_{1,t}$, $z_{2,t}$, $z_{3,t}$ and $z_{4,t}$. Given this result, (C.1), (C.7), (C.11) and (C.12) together represent the dynamical system of the model.

To linearize the model around the steady-state equilibrium, we slightly re-organize the system as equations of $\{z_{1,t}, z_{2,t}, z_{3,t}, z_{4,t}, \omega_t\}$. First, combining (C.18) and (C.21) yields

$$\frac{z_{2,t}}{z_{3,t}} = \frac{\mu^S(1-l_{x,t}^N)}{\mu^N(1-l_{x,t}^S)} (z_{4,t})^{\frac{1}{\sigma}} (1-\alpha)^{\frac{\sigma-1}{\sigma}} \alpha^{\frac{1-\sigma}{\sigma}} \delta^{\frac{1-\sigma}{\sigma}} \left(\frac{l_{x,t}^N}{l_{x,t}^S}\right)^{-\frac{1}{\sigma}}. \tag{C.22}$$

Then substituting (C.14) into (C.22) gives rise to

$$\frac{l_{r,t}^S}{l_{r,t}^N} = \frac{z_{3,t}}{z_{2,t}} \left(\frac{1-\alpha}{\alpha}\right) \omega_t. \tag{C.23}$$

By rearranging (C.18), we can show that

$$z_{2,t} = \frac{\beta}{\mu^N z_{1,t}} \left(\frac{z_{4,t}}{1+z_{4,t}}\right)^{\frac{1}{\sigma}} \left(\frac{\Xi}{l_{x,t}^N}\right)^{\frac{1-\sigma}{\sigma}} (l_{x,t}^N)^{\frac{1-\sigma}{\sigma}} (1-\alpha)^{-\frac{1}{\sigma}} (l_{x,t}^N)^{-\frac{1}{\sigma}}$$
$$= \frac{\beta}{\mu^N z_{1,t}} \left(\frac{z_{4,t}}{1+z_{4,t}}\right)^{\frac{1}{\sigma}} [\Xi(\omega_t)]^{\frac{1-\sigma}{\sigma}} (1-\alpha)^{-\frac{1}{\sigma}} (l_{x,t}^N)^{-1}, \tag{C.24}$$

where

$$\Xi(\omega_t) = \left\{(1-\alpha)^{\frac{\sigma-1}{\sigma}} \left(\frac{z_{4,t}}{1+z_{4,t}}\right)^{\frac{1}{\sigma}} + (\delta\alpha)^{\frac{\sigma-1}{\sigma}} \left[\left(\frac{\alpha}{1-\alpha}\right) \delta^{1-\sigma} \left(\frac{\mu^S}{\mu^N}\right) \omega_t^{-\sigma} z_{4,t}\right]^{\frac{1-\sigma}{\sigma}} \left(\frac{1}{1+z_{4,t}}\right)^{\frac{1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}.$$

By using (C.14) to replace $l_{x,t}^S$ and (C.24) to replace $l_{x,t}^N$ in (C.23), we can rewrite (C.23) such that

$$\begin{aligned}\frac{z_{3,t}}{z_{2,t}}\frac{1-\alpha}{\alpha}\omega_t &= \frac{1-\left[\left(\frac{\alpha}{1-\alpha}\right)^{-1}\delta^{\sigma-1}\left(\frac{\mu^S}{\mu^N}\right)^{-\sigma}\omega_t^{\sigma}z_{4,t}^{-1}\right]l_{x,t}^N}{1-l_{x,t}^N}\\ &= \frac{1-\left[\left(\frac{\alpha}{1-\alpha}\right)^{-1}\delta^{\sigma-1}\left(\frac{\mu^S}{\mu^N}\right)^{-\sigma}\omega_t^{\sigma}z_{4,t}^{-1}\right]\frac{\beta}{\mu^N z_{1,t}z_{2,t}}\left(\frac{z_{4,t}}{1+z_{4,t}}\right)^{\frac{1}{\sigma}}\left[\Xi(\omega_t)\right]^{\frac{1-\sigma}{\sigma}}(l_{x,t}^N)^{\frac{1-\sigma}{\sigma}}(1-\alpha)^{-\frac{1}{\sigma}}}{1-\frac{\beta}{\mu^N z_{1,t}z_{2,t}}\left(\frac{z_{4,t}}{1+z_{4,t}}\right)^{\frac{1}{\sigma}}\left[\Xi(\omega_t)\right]^{\frac{1-\sigma}{\sigma}}(1-\alpha)^{-\frac{1}{\sigma}}}.\end{aligned} \tag{C.25}$$

This equation implicitly solves ω_t as a function of the four transformed variables $\{z_{1,t}, z_{2,t}, z_{3,t}, z_{4,t}\}$. Similarly, using (C.14) and (C.23) to substitute for $l_{x,t}^S$ and $l_{x,t}^N$, we can rewrite the four differential equations of $\{z_{1,t}, z_{2,t}, z_{3,t}, z_{4,t}\}$ such that

$$\frac{\dot{z}_{1,t}}{z_{1,t}} = g_L - (1-\xi)(\kappa-1)(1-\alpha)\left\{\frac{z_{1,t}}{\beta} - \frac{1}{\mu^N z_{2,t}}\left(\frac{z_{4,t}}{1+z_{4,t}}\right)^{\frac{1}{\sigma}}\left[\Xi(\omega_t)\right]^{\frac{1-\sigma}{\sigma}}(1-\alpha)^{-\frac{1}{\sigma}}\right\}, \tag{C.26}$$

$$\frac{\dot{z}_{2,t}}{z_{2,t}} = \rho - g_L + \frac{1-\alpha}{\beta}z_{1,t}\left\{1-\left[\mu^N + \frac{1}{z_{4,t}}(\mu^N-1)\right]\frac{\beta}{\mu^N z_{1,t}z_{2,t}}\left(\frac{z_{4,t}}{1+z_{4,t}}\right)^{\frac{1}{\sigma}}\left[\Xi(\omega_t)\right]^{\frac{1-\sigma}{\sigma}}(1-\alpha)^{-\frac{1}{\sigma}}\right\}, \tag{C.27}$$

$$\begin{aligned}\frac{\dot{z}_{3,t}}{z_{3,t}} &= \rho - g_L + \frac{1-\alpha}{\beta}z_{1,t}\left\{1-\frac{\beta}{\mu^N z_{1,t}z_{2,t}}\left(\frac{z_{4,t}}{1+z_{4,t}}\right)^{\frac{1}{\sigma}}\left[\Xi(\omega_t)\right]^{\frac{1-\sigma}{\sigma}}(1-\alpha)^{-\frac{1}{\sigma}}\right\}\\ &\quad - \frac{\alpha(\mu^S-1)(1+z_{4,t})\left[\left(\frac{\alpha}{1-\alpha}\right)^{-1}\delta^{\sigma-1}\left(\frac{\mu^S}{\mu^N}\right)^{-\sigma}\omega_t^{\sigma}z_{4,t}^{-1}\right]}{\gamma}z_{1,t},\\ &\quad + \frac{(\mu^N-1)(1-\alpha)\omega_t\frac{1+z_{4,t}}{z_{4,t}}\left\{\frac{\beta}{\mu^N z_{1,t}z_{2,t}}\left(\frac{z_{4,t}}{1+z_{4,t}}\right)^{\frac{1}{\sigma}}\left[\Xi(\omega_t)\right]^{\frac{1-\sigma}{\sigma}}(1-\alpha)^{-\frac{1}{\sigma}}\right\}}{\gamma}z_{1,t},\end{aligned} \tag{C.28}$$

and

$$\begin{aligned}\frac{\dot{z}_{4,t}}{z_{4,t}} &= \left\{\frac{(1-\alpha)z_{1,t}}{\beta}\left(\kappa-1+\frac{\kappa}{z_{4,t}}\right) - (1+z_{4,t})\frac{\alpha z_{1,t}}{\gamma}\left(1+\frac{1}{z_{4,t}}\right)\left[\frac{z_{3,t}}{z_{2,t}}\frac{1-\alpha}{\alpha}\omega_t\right]\right\}\\ &\quad\times\left\{1-\frac{\beta}{\mu^N z_{1,t}z_{2,t}}\left(\frac{z_{4,t}}{1+z_{4,t}}\right)^{\frac{1}{\sigma}}\left[\Xi(\omega_t)\right]^{\frac{1-\sigma}{\sigma}}(1-\alpha)^{-\frac{1}{\sigma}}\right\}.\end{aligned} \tag{C.29}$$

Due to its complexity, we resort to *Mathematica* for linearization and compute the eigenvalues of the Jacobian matrix. The matrix takes the form of

$$\begin{bmatrix}\dot{z}_{1,t}\\ \dot{z}_{2,t}\\ \dot{z}_{3,t}\\ \dot{z}_{4,t}\end{bmatrix} = \begin{bmatrix}J_{11} & J_{12} & J_{13} & J_{14}\\ J_{21} & J_{22} & J_{23} & J_{24}\\ J_{31} & J_{32} & J_{33} & J_{34}\\ J_{41} & J_{42} & J_{43} & J_{44}\end{bmatrix}\begin{bmatrix}z_{1,t}-z_1^*\\ z_{2,t}-z_2^*\\ z_{3,t}-z_3^*\\ z_{4,t}-z_4^*\end{bmatrix}$$

with

$$J_{ij} = \frac{\partial \dot{z}_{i,t}}{\partial z_{j,t}}\Big|_{(z_1^*, z_2^*, z_3^*, z_4^*, \omega^*)} + \frac{\partial \dot{z}_{i,t}}{\partial \omega_t}\frac{\partial \omega_t}{\partial z_{j,t}}\Big|_{(z_1^*, z_2^*, z_3^*, z_4^*, \omega^*)},$$

where $\{i, j\} = \{1, 2, 3, 4\}$. Substituting the numerical values in our calibration into the Jacobian matrix yields

$$J = \begin{bmatrix} -0.1677 & -0.0610 & 0.0079 & 0.0701 \\ 0.4407 & 0.6242 & -0.0535 & -0.2830 \\ 0.8073 & -0.5173 & 0.4355 & -0.9990 \\ -3.3417 & -1.5475 & 0.0077 & -3.6304 \end{bmatrix}.$$

In this case, the eigenvalues for this Jacobian matrix are given by $\{-3.6737, 0.7192, 0.3880, -0.1719\}$, respectively. Therefore, the dynamics of this equation system is characterized by saddle-path stability, implying that the global economy is on a unique and stable balanced growth path.

Appendix D : The variety-expansion model

To examine the robustness of our results, we consider an extension in which the process of innovation is variety expansion, following the framework of Gustafsson and Segerstrom (2011). The notations for the variables in the baseline model remain the same as their parallel counterparts in this extended model. Moreover, we focus on the steady-state equilibrium.

The production function of final goods is altered to

$$Y_t = \left\{ \int_0^{m_t} [x_t(j)]^{\frac{\sigma-1}{\sigma}} dj \right\}^{\frac{\sigma}{\sigma-1}}, \tag{D.1}$$

where m_t is the number of intermediate-good varieties. Therefore, the demand function of $x_t(j)$ is given by

$$x_t(j) = \frac{Y_t}{[p_t(j)]^{\sigma}}, \tag{D.2}$$

where the price of final goods $P_t \equiv \{\int_0^{m_t} [p_t(j)]^{1-\sigma} dj\}^{1/(1-\sigma)}$ is normalized to unity. In this variety-expansion framework, new products are invented and manufactured in the North, whereas old products remain being manufacturing in the South once the technology of manufacturing these products has been transferred. Therefore, the product cycle becomes one-way, implying that Assumption **??** no longer applies. The production functions and price strategies remain unchanged, with patent breadth in each country determining the monopolistic prices. The profits for a typical Northern monopolist and a Southern affiliate for any variety $j \in [0, m_t]$ are given by

$$\pi_t^N = p_t^N x_t^N(j) - w_t^N L_{x,t}^N(j) = (\mu^N - 1) w_t^N \frac{Y_t}{(\mu^N w_t^N)^{\sigma}} \tag{D.3}$$

and

$$\pi_t^S = p_t^S x_t^S(j) - w_t^S L_{x,t}^S(j) = (\mu^S - 1) \frac{w_t^S}{\delta} \frac{Y_t}{(\mu^S w_t^S / \delta)^{\sigma}}, \tag{D.4}$$

respectively, given that the monopolistic price and profit flow are identical across varieties. Moreover, Assumption 2 still holds to ensure that Northern monopolists are willing to shift their production to the South, namely $\pi_t^S > \pi_t^N$ holds.

To develop a new product variety, a representative Northern firm i devotes β / m_t^{ξ} units of labor in innovative R&D activities. The flow of new products developed at time t is given by $\dot{m}_t(i) = m_t^{\xi} L_{r,t}^N(i) / \beta$. Thus, the aggregate flow of new products developed in the North is

$$\dot{m}_t = \frac{m_t^{\xi} L_{r,t}^N}{\beta}, \tag{D.5}$$

where $L_{r,t}^N \equiv \int L_{r,t}^N(i) di$ denotes the aggregate R&D labor in the North. Free entry into R&D yields the zero-expected-profit condition for innovative R&D such that

$$v_t^N \dot{m}_t(i) = w_t^N L_{r,t}^N(i) \Leftrightarrow v_t^N = \beta w_t^N m_t^{-\xi}. \tag{D.6}$$

Similarly, the number of varieties transferred to the South by the foreign affiliate of a Northern monopolist is given by $\dot{m}_t^S(i) = m_t^{\xi} L_{r,t}^S(i)/\gamma$. Thus, the aggregate flow of varieties shifted to the South is

$$\dot{m}_t^S = \frac{m_t^{\xi} L_{r,t}^S}{\gamma}, \tag{D.7}$$

where $L_{r,t}^S \equiv \int L_{r,t}^S(i)di$ denotes the aggregate R&D labor in the South. Moreover, denote by $\lambda^S = \dot{m}_t^S/m_t$ the rate of technology transfer. Free entry into R&D yields the zero-expected-profit condition for adaptive R&D such that

$$(v_t^S - v_t^N)\dot{m}_t^S(i) = w_t^S L_{r,t}^S(i) \Leftrightarrow v_t^S - v_t^N = \gamma w_t^S m_t^{-\xi}. \tag{D.8}$$

In addition, the no-arbitrage conditions that determine the values of v_t^N and v_t^S are, respectively, given by

$$r_t v_t^N = \pi_t^N + \dot{v}_t^N, \tag{D.9}$$

and

$$r_t v_t^S = \pi_t^S + \dot{v}_t^S. \tag{D.10}$$

Next, we consider the labor-market-clearing conditions in the North and the South. The Northern labor-market-clearing condition is given by

$$\int_{m_t^N} L_{x,t}^N(j)dj + L_{r,t}^N = \theta_t^N m_t \overline{L}_{x,t}^N + \beta g m_t^{1-\xi} = L_t^N \Leftrightarrow \theta_t^N m_t \frac{\overline{L}_{x,t}^N}{L_t^N} + \beta g \Phi_t = 1, \tag{D.11}$$

where $\theta_t^N \equiv m_t^N/m_t$ is the share of varieties produced in the North, $\overline{L}_{x,t}^N$ is the average Northern production labor, and $\Phi_t \equiv m_t^{1-\xi}/L_t^N$ is the average productivity per Northern worker. The Southern labor-market-clearing condition is given by

$$\int_{m_t^S} L_{x,t}^S(j)dj + L_{r,t}^S = \theta_t^S m_t \overline{L}_{x,t}^S + \gamma\lambda^S m_t^{1-\xi} = L_t^S \Leftrightarrow \theta_t^S m_t \frac{\overline{L}_{x,t}^S}{L_t^S} + \gamma\lambda^S \Phi_t \frac{1-\alpha}{\alpha} = 1, \tag{D.12}$$

where $\theta_t^S \equiv m_t^S/m_t$ is the share of varieties produced in the South and $\overline{L}_{x,t}^S$ is the average Southern production labor.

In the steady-state equilibrium, $\Phi_t = \Phi$ must be constant, which yields the growth rate of the number of varieties such that $g = \dot{m}_t/m_t = g_L/(1-\xi)$, and the condition such that $\dot{m}_t^S/m_t^S = \dot{m}_t^N/m_t^N = \dot{m}_t/m_t$. In addition, according to the rate of technology transfer $\lambda^S = \dot{m}_t^S/m_t$, we obtain the steady-state shares of varieties in the North and the South as follows:

$$\theta_t^S = \frac{\lambda^S}{g}; \qquad \theta_t^N = \frac{g-\lambda^S}{g}. \tag{D.13}$$

To derive the steady-state innovative R&D condition, we take the log of (D.6) and differentiate the resulting equation to obtain $\dot{v}_t^N/v_t^N = (1-\xi)g$. Substituting this equation into (D.9) yields $v_t^N = \pi^N/(\rho+\xi g)$. Then, combining (D.3) and (D.6) with the above equation yields the steady-state innovative R&D condition such that

$$(\mu^N - 1)\frac{m_t \overline{L}_{x,t}^N}{L_t^N} = \beta\Phi(\rho + \xi g). \tag{D.14}$$

Similarly, from (D.8) we have $\dot{v}_t^S / v_t^S = (1-\xi)g$. Substituting it into (D.10) yields $v_t^S = \pi_t^S/(\rho + \xi g)$. Combining this equation with (D.4) and (D.6) yields the steady-state adaptive R&D condition such that

$$(\mu^S - 1)\frac{\overline{L}_{x,t}^S m_t}{L_t^N} - (\mu^N - 1)\omega\frac{\overline{L}_{x,t}^N m_t}{L_t^N} = \gamma(\rho + \xi g)\Phi. \tag{D.15}$$

Then, combining (D.11) and (D.14) generates the Northern steady-state condition such that

$$\beta\Phi\left[\frac{(\rho + \xi g)(g - \lambda^S)}{g(\mu^N - 1)} + g\right] = 1. \tag{D.16}$$

This equation is the counterpart condition against (46) in the sense that Φ is positively correlated with λ^S. Additionally, the Southern steady-state condition is obtained by combining (D.11), (D.12), (D.14), and (D.15), which is given by

$$\frac{\Phi\lambda^S(1-\alpha)}{\alpha g}\left[\frac{\delta(\rho + \xi g)(\gamma + \beta\omega)}{\mu^S - 1} + \gamma g\right] = 1, \tag{D.17}$$

which is also qualitatively similar to (47) in the sense that Φ is negatively correlated with λ^S. In this case, equations (D.16) and (D.17) together determine a unique steady-state solution of two unknowns $\{\Phi, \lambda^S\}$, which is graphically illustrated as in Figure 1. Furthermore, dividing (D.14) by (D.15) leads to the equation that pins down the steady-state relative wage rate, which is identical to (54). Therefore, the static analysis of a change in μ^N or μ^S on the rate of relative wage ω, the rate of international technology transfer λ^S, and the temporary innovation rate in the North are robust to the counterparts in the baseline model.

Appendix E : The model with trade costs

In this appendix, we extend the baseline model by incorporating positive trade costs between the countries. Specifically, we follow Gustafsson and Segerstrom (2010) to assume that trade costs take the "iceberg" form such that $\tau > 1$ unites of a product must be produced and exported in order to have one unit arriving at its destination. Again, we only illustrate the main parts that are changed in this extended model.

To simplify the analysis, we assume that households consume differentiated goods produced in both countries. The discounted lifetime utility of a typical household in economy $i \in \{N, S\}$ is given by

$$U^i \equiv \int_0^\infty e^{-(\rho-g_L)t} \ln u^i(t) dt, \tag{E.1}$$

where the static utility is given by

$$u_t^i = \left\{ \int_0^1 [x_t^i(j)]^{\frac{\sigma-1}{\sigma}} dj \right\}^{\frac{\sigma}{\sigma-1}}. \tag{E.2}$$

Solving the consumer's utility-maximization problem yields the individual consumer's demand function such that

$$x_t^i(j) = \frac{[p_t^i(j)]^\sigma E_t^i}{(P_t^i)^{1-\sigma}}, \tag{E.3}$$

where E_t^i is the per capita consumer expenditure in country i at time t and $P_t^i \equiv \{\int_0^1 [p_t^i(j)]^{1-\sigma}\}^{1/(1-\sigma)}$ is an index of consumer prices. Maximizing (E.1) subject to (E.2) where (E.3) has been used to substitute for $x_t^i(j)$ yields the Euler equation such that

$$\frac{\dot{E}_t^N}{E_t^N} = \frac{\dot{E}_t^S}{E_t^S} = r_t - \rho. \tag{E.4}$$

In this extension, we focus on the steady-state equilibrium where both wage rates and consumption expenditures are stationary over time (i.e., $w_t^N = w^N$, $w_t^S = w^S$, $E_t^N = E^N$, and $E_t^S = E^S$) , so using (E.4) implies that $r_t = \rho$. In addition, the Southern wage rate is chosen as the numeraire price ($w_t^S = 1$), so all prices are measured relative to the price of Southern labor. Then the wage rate of the North relative to the South is given by $\omega = w^N$.

In the presence of trade costs, Northern consumers face different prices than Southern consumers. This alters the pricing strategy of Northern quality leaders and Southern affiliates. The condition $\omega\delta > 1$ is changed to $\omega\delta > \tau$ to take into account the additional transportation cost, and Assumption 1 is altered to $\omega\delta < \tau(z/\mu^N)$ for ensuring the two-product cycle.

To derive the profits of Northern leaders and Southern affiliates, we first show the demand functions of Northern and Southern consumers. On the one hand, the demand of a Northern consumer for a domestically produced product is $x_t^{Nd} = [p_t^{Nd}(j)]^{-\sigma} E^N / (P_t^N)^{1-\sigma}$ and the demand for an imported good (exported by the South) is $x_t^{S*} = [p_t^{S*}(j)]^{-\sigma} E^N / (P_t^N)^{1-\sigma}$. Thus, the Northern price index is given by $P_t^N = \left\{ \int_{\theta_t^N} [p_t^{Nd}(j)]^{1-\sigma} + \int_{\theta_t^S} [p_t^{S*}(j)]^{1-\sigma} \right\}^{1/(1-\sigma)}$. On the other hand, the demand of a Southern consumer for a domestically produced product is $x_t^{Sd} =$

$[p_t^{Sd}(j)]^{-\sigma}E^S/(P_t^S)^{1-\sigma}$ and the demand for an imported good (exported by the North) is $x_t^{N*} = [p_t^{N*}(j)]^{-\sigma}E^S/(P_t^S)^{1-\sigma}$. Thus, the Southern price index is $P_t^S = \left\{\int_{\theta_t^N}[p_t^{N*}(j)]^{1-\sigma} + \int_{\theta_t^S}[p_t^{Sd}(j)]^{1-\sigma}\right\}^{1/(1-\sigma)}$.

In the presence of trade costs, the Northern quality leader in industry $j \in [0,1]$ sets the domestic monopolistic price $p_t^{Nd}(j)$ to $\mu^N\omega/[z^{n_t(j)}]$ and the export price $p_t^{N*}(j)$ to $\tau\mu^N\omega/[z^{n_t(j)}]$. Given households' demand functions, the monopolistic profit of a Northern quality leader comprises of domestic and export profits such that

$$\pi_t^N(j) = (\mu^N - 1)\frac{\omega q_t(j)}{(\mu^N\omega)^\sigma}\left[\frac{\tau^{1-\sigma}L_t^S E^S}{(P_t^S)^{1-\sigma}} + \frac{L_t^N E^N}{(P_t^N)^{1-\sigma}}\right]. \tag{E.5}$$

The Southern affiliate sets the domestic monopolistic price $p_t^{Sd}(j)$ to $\mu^S/(\delta z^{n_t(j)})$ and the export price $p_t^{S*}(j)$ to $\tau\mu^S/(\delta z^{n_t(j)})$. Given households' demand functions, the monopolistic profit for a Southern affiliate is given by

$$\pi_t^S(j) = (\mu^S - 1)\frac{q_t(j)}{\delta}\left(\frac{\delta}{\mu^S}\right)^\sigma\left[\frac{\tau^{1-\sigma}L_t^N E^N}{(P_t^N)^{1-\sigma}} + \frac{L_t^S E^S}{(P_t^S)^{1-\sigma}}\right]. \tag{E.6}$$

Moreover, the labor demands for an average-quality product produced by a Northern leader is

$$\overline{L}_{x,t}^N = \int_0^1 L_{x,t}^N(j)dj = \frac{\tau L_t^S E^S \int_0^1 q_t(j)dj}{(P_t^S)^{1-\sigma}(\mu^N\omega\tau)^\sigma} + \frac{L_t^N E^N \int_0^1 q_t(j)dj}{(P_t^N)^{1-\sigma}(\mu^N\omega)^\sigma} = \frac{Q_t}{(\mu^N\omega)^\sigma}\left[\frac{\tau^{1-\sigma}L_t^S E^S}{(P_t^S)^{1-\sigma}} + \frac{L_t^N E^N}{(P_t^N)^{1-\sigma}}\right], \tag{E.7}$$

whereas the labor demands for an average-quality product produced by a Southern affiliate is

$$\overline{L}_{x,t}^S = \int_0^1 L_{x,t}^S(j)dj = \frac{\tau L_t^N E^N \int_0^1 q_t(j)dj}{\delta(P_t^N)^{1-\sigma}(\mu^S\tau/\delta)^\sigma} + \frac{L_t^S E^S \int_0^1 q_t(j)dj}{\delta(P_t^S)^{1-\sigma}(\mu^S/\delta)^\sigma} = \frac{Q_t}{\delta}\left(\frac{\delta}{\mu^S}\right)^\sigma\left[\frac{\tau^{1-\sigma}L_t^N E^N}{(P_t^N)^{1-\sigma}} + \frac{L_t^S E^S}{(P_t^S)^{1-\sigma}}\right]. \tag{E.8}$$

Using these equations, the labor demands for product j are expressed as

$$L_{x,t}^N(j) = \frac{q_t(j)}{Q_t}\overline{L}_{x,t}^N; \quad L_{x,t}^S(j) = \frac{q_t(j)}{Q_t}\overline{L}_{x,t}^S. \tag{E.9}$$

The behaviors of innovative and adaptive R&D sectors follow exactly those in the baseline setup. Following the derivation in the baseline model, we can show that equations (17)–(47) continue to hold. Therefore, in this extension of positive trade costs, there still exists a unique steady-state equilibrium pinning down the values of Φ and λ^S, given that the steady-state relative wage ω is a function of the patent instruments $\{\mu^N, \mu^S\}$. However, the steady-state relative wage equation (54) is changed. From (E.7)–(E.8), we obtain

$$\frac{\overline{L}_{x,t}^N}{\overline{L}_{x,t}^S} = \delta^{\sigma-1}\left(\frac{\mu^N}{\mu^S}\right)^{-\sigma}\omega^{-\sigma}\left[\frac{\frac{\tau^{1-\sigma}L_t^S E^S}{(P_t^S)^{1-\sigma}} + \frac{L_t^N E^N}{(P_t^N)^{1-\sigma}}}{\frac{\tau^{1-\sigma}L_t^N E^N}{(P_t^N)^{1-\sigma}} + \frac{L_t^S E^S}{(P_t^S)^{1-\sigma}}}\right], \tag{E.10}$$

Combining (44) and (45), together with (E.10), yields the equation determining the steady-state relative wage rate such that

$$\frac{\gamma}{\beta\omega^{\sigma}} + \omega^{1-\sigma} = \delta^{1-\sigma}\left(\frac{\mu^N}{\mu^S}\right)^{\sigma}\frac{\mu^S-1}{\mu^N-1}\left[\frac{\frac{\tau^{1-\sigma}L_t^S E^S}{(P_t^S)^{1-\sigma}} + \frac{L_t^N E^N}{(P_t^N)^{1-\sigma}}}{\frac{\tau^{1-\sigma}L_t^N E^N}{(P_t^N)^{1-\sigma}} + \frac{L_t^S E^S}{(P_t^S)^{1-\sigma}}}\right] \tag{E.11}$$

where

$$\begin{aligned} P_t^N &= \left\{\int_{\theta_t^N}[p_t^{Nd}(j)]^{1-\sigma}dj + \int_{\theta_t^S}[p_t^{N*}(j)]^{1-\sigma}dj\right\}^{\frac{1}{1-\sigma}} = \left\{\int_{\theta_t^N}\left[\frac{\mu^N\omega}{z^{n_t(j)}}\right]^{1-\sigma}dj + \int_{\theta_t^S}\left[\frac{\tau\mu^N\omega}{z^{n_t(j)}}\right]^{1-\sigma}dj\right\}^{\frac{1}{1-\sigma}} \\ &= \left(Q_t^N + \tau^{1-\sigma}Q_t^S\right)^{\frac{1}{1-\sigma}}(\mu^N\omega), \end{aligned} \tag{E.12}$$

$$\begin{aligned} P_t^S &= \left\{\int_{\theta_t^N}[p_t^{S*}(j)]^{1-\sigma}dj + \int_{\theta_t^S}[p_t^{Sd}(j)]^{1-\sigma}dj\right\}^{\frac{1}{1-\sigma}} = \left\{\int_{\theta_t^N}\left[\frac{\mu^S\omega}{\delta z^{n_t(j)}}\right]^{1-\sigma}dj + \int_{\theta_t^S}\left[\frac{\tau\mu^S\omega}{\delta z^{n_t(j)}}\right]^{1-\sigma}dj\right\}^{\frac{1}{1-\sigma}} \\ &= \left(\tau^{1-\sigma}Q_t^N + Q_t^S\right)^{\frac{1}{1-\sigma}}\left(\frac{\mu^S\omega}{\delta}\right). \end{aligned} \tag{E.13}$$

Define by $\eta \equiv (L_t^N E^N)/(L_t^S E^S)$ the aggregate consumption expenditure of Northern households relative to Southern households. Then (E.11) can be rewritten as

$$\frac{\gamma}{\beta\omega^{\sigma}} + \omega^{1-\sigma} = \delta^{1-\sigma}\left(\frac{\mu^N}{\mu^S}\right)^{\sigma}\frac{\mu^S-1}{\mu^N-1}\underbrace{\left[\frac{\frac{\tau^{1-\sigma}/\eta}{\left(\tau^{1-\sigma}Q_t^N/Q_t+Q_t^S/Q_t\right)(\mu^S/\delta)^{1-\sigma}} + \frac{1}{(Q_t^N/Q_t+\tau^{1-\sigma}Q_t^S/Q_t)(\mu^N\omega)^{1-\sigma}}}{\frac{\tau^{1-\sigma}}{(Q_t^N/Q_t+\tau^{1-\sigma}Q_t^S/Q_t)(\mu^N\omega)^{1-\sigma}} + \frac{1/\eta}{(\tau^{1-\sigma}Q_t^N/Q_t+Q_t^S/Q_t)(\mu^S/\delta)^{1-\sigma}}}\right]}_{\Omega} \tag{E.14}$$

Comparing (E.14) to (54), it can be seen that the only difference comes from the aggregate term Ω in the bracket. When $\tau = 1$ (i.e., a zero transportation cost), (E.14) is reduced to (54). In this generalized version, (46), (47) and (E.14) together constitute a system of equations that solves for three endogenous variables $\{\Phi, \lambda^S, \omega\}$.

Therefore, the static analysis of a change in μ^N or μ^S on the rate of relative wage ω, the rate of international technology transfer λ^S, and the temporary innovation rate in the North are still robust to the counterparts in the baseline model. More importantly, a decrease in trade costs τ leads to no changes in the rate of international technology transfer λ^S and the temporary innovation rate in the North, but an ambiguous impact on rate of relative wage ω. Specifically, making use of (E.11), it can be shown that a decrease in τ leads to a permanent increase (decrease) in ω if the North is larger (smaller) than the South in terms of purchasing power (i.e., $L_t^N E^N/(P_t^N)^{1-\sigma} > (<) L_t^S E^S/(P_t^S)^{1-\sigma}$).

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