Inflation and Income Inequality in a Schumpeterian Economy with Heterogeneous Wealth and Skills*†

Zhijie Zheng
Beijing Normal University

Chien-Yu Huang
Nanjing Audit University

Xi Wan
Nanjing Audit University

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Abstract

We explore the effects of monetary policy on innovation and income inequality in a scale-invariant endogenous growth model with heterogeneous households and a cash-in-advance (CIA) constraint on R&D investment. Household heterogeneity arises from unequal distributions of wealth and skill, giving rise to the interest income and labor income inequalities, respectively. We find that inflation unambiguously reduces innovation and economic growth, whereas its impact on income inequality can be positive, negative, or U-shaped. The relation between inflation and income inequality depends on the relative dominance of wealth heterogeneity to skill heterogeneity, and how the ratio of interest income to labor income responds to inflation. Moreover, the model is calibrated to the US economy and the numerical results support these implications on income inequality.

JEL classification: D31, O30; O40; E41.

Keywords: Income inequality; Inflation; Wealth and skill heterogeneity; Endogenous economic growth.

*Zhijie Zheng: Center for Innovation and Development Studies, Beijing Normal University, 519087, China (e-mail: zhengzhijie1919@gmail.com); Chien-Yu Huang: Institute of Urban Development, Nanjing Audit University, Nanjing, Jiangsu, 211815, China (e-mail: chuang4@ncsu.edu); Xi Wan: International Joint Audit Institute, Belt and Road National Audit Research Center, Nanjing Audit University, Nanjing, Jiangsu, 211815, China (e-mail: xiiwan@gmail.com).

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1 Introduction

It is widely acknowledged that people in an economy reap unequal benefits of economic growth (e.g., Kuznets (1955), Alesina and Rodrik (1994), and Piketty (2014)). An important question centers on how policy instruments, which aim to promote economic growth, exacerbate or mitigate income inequality? A few recent studies, such as Chu *et al.* (2019) and Zheng (2020), have started to investigate the impacts of monetary policy on innovation, economic growth, and income inequality. However, these studies mainly model household heterogeneity by allowing households' different levels of wealth and regard wealth heterogeneity as the source of income inequality. Our study departs from the literature by introducing both wealth and skill heterogeneities that govern the interest income and labor income inequalities, respectively, and finds that this feature leads to novel results.[1]

The model we construct has three main features. First, we follow García-Peñalosa and Turnovsky (2006) and Chu and Cozzi (2018) in treating the unequal wealth distribution as an important driving force of income inequality. This approach is motivated by the empirical evidence in Piketty (2014). The assumption of different wealth endowments gives rise to an unequal distribution of households' interest income, in part causing income inequality. Second, our model also considers skill heterogeneity, and as a result households' labor income distribution is unequal as well. The inclusion of skill heterogeneity is also motivated by an increasing number of empirical studies, such as Castelló and Doménech (2002), Castelló-Climent (2010) and Hasanov and Izraeli (2011), who have documented a high correlation between skill/human capital inequality and income inequality.[2] Third, we follow the tradition in a large literature (e.g., Chu and Cozzi (2014), He and Zou (2016), and Huang *et al.* (2017)) to introduce money demand in a Schumpeterian growth model by imposing a cash-in-advance (CIA) constraint on entrepreneurs' R&D activities.[3]

Within this growth-theoretic framework, we explore the effects of monetary policy on innovation, economic growth, and income inequality, respectively. First, we find that inflation unambiguously stifles innovation and economic growth. Intuitively, a higher inflation rate raises the cost of innovating activities via the CIA constraint on R&D, which in turn decreases the arrival rate of innovation. As a result, the growth rate of technology and output decreases.

Our second main finding is that inflation can generate a positive, negative, or U-shaped effect

[1]Some other studies in the literature of inequality and growth have considered skill/human capital inequality from the aspect of education, such as Loury (1981), Glomm and Ravikumar (1992), Bénabou (2002) and Basu and Getachew (2015). These papers, however, do not decompose income inequality into interest and labor income inequalities as in our paper.

[2]More specifically, Castelló and Doménech (2002), Checchi (2004), and Castelló-Climent (2010) report that the human capital inequality is significantly correlated with income inequality in a cross-country perspective. Furthermore, Rodríguez-Pose and Tselios (2009) and Hasanov and Izraeli (2011) show that high levels of inequality in educational attainment are associated with high income inequality across regions of the E.U. and the US.

[3]Chu *et al.* (2015) provide a detailed discussion of modeling money demand using this approach. Other studies that adopt this approach include Chu *et al.* (2017), Gil and Iglésias (2020), Huang *et al.* (2021), and Zheng *et al.* (2021).

on income inequality. In our model, income distribution is a "weighed" combination of wealth and skill distributions. Inflation affects income inequality by changing the "relative weight" of wealth heterogeneity and skill heterogeneity, which is governed by the ratio of interest income to labor income. Specifically, the impact of inflation on the ratio can be decomposed into three channels. First, a higher inflation rate reduces the economic growth rate and the equilibrium real interest rate, which decreases the return rate of wealth. This *interest-rate* effect leads to a lower ratio of interest income to labor income. Second, by lowering down the rate of creative destruction, inflation increases the market value of monopolistic firms. This *asset-value* in turn raises the value of financial assets held by households and tends to increase the ratio. Third, the suppressed R&D expenditure due to inflation implies lower demand for cash flow and lower bond value held by households. This *bond-value* effect tends to decrease the ratio of interest income to labor income. Combining these effects yields an overall positive effect of inflation on the ratio of interest income to labor income under a wide range of plausible parameters.[4] Accordingly, when wealth heterogeneity dominates skill heterogeneity, a rise in inflation exacerbates income inequality because it raises the contribution/weight of wealth heterogeneity relative to skill heterogeneity. In contrast, when wealth heterogeneity is dominated by skill heterogeneity, a rise in inflation may generate a decreasing or U-shaped effect on income inequality.[5]

Our study thus provides a novel mechanism that partially reconciles the mixed empirical evidence on the nexus between inflation and income inequality. For example, studies by Edwards (1997), Albanesi (2007) and Ghossoub and Reed (2017) observe a positive linkage between inflation and income inequality, whereas Jäntti (1994) and Mocan (1999) document a negative relation instead. Interestingly, some recent empirical studies argue that the nexus is contingent on the level of the inflation rate. For instance, Galli and van der Hoeven (2001) and Balcilar *et al.* (2018) report a U-shaped relation between inflation and income inequality in OECD countries and the US, respectively.[6] In addition, we calibrate the model to the US economy and perform quantitative analysis to evaluate the effects of inflation on growth and income inequality. We find that the quantitative results support our theoretical predictions and these results are robust to targeted empirical moments and parameters.

Our study is closely related to the literature on inflation and income inequality within an innovation-driven growth model. By building up a Schumpeterian growth model with random quality improvements, Chu *et al.* (2019) find an inverted-U effect of inflation on income inequality. In their study, the inverted-U real interest rate effect of inflation, arising from the inverted-U

[4] Inflation can also generate a U-shaped effect on the ratio of interest income to labor income if the intermediate-goods share of output is sufficiently small. As shown in Section 4, the positive, negative or U-shaped effects of inflation on income inequality still hold in this circumstance.

[5] As shown in Section 3, the U-shaped relation between inflation and income inequality arises because inflation alters the relative heterogeneity between wealth and skills. Therefore, there may exist a threshold for the wealth-skill relative heterogeneity below (above) which a higher inflation rate mitigates (enlarges) income inequality.

[6] An exceptional result comes from Chu *et al.* (2019), who find the overall effect of inflation on income inequality follows an inverted-U pattern.

growth effect of inflation, always dominates the other financial assets value effect. This therefore causes inflation to have an inverted-U impact on the ratio of interest income to labor income. The relation between inflation and income inequality then follows a hump-shaped pattern, as income inequality in their study comes from the unequal distribution of wealth endowments. Zheng (2020) and Zheng *et al.* (2020) address a similar question in growth models with quality improvement and variety expansion, respectively, in which firms suffer from costly pricing adjustment.[7] Zheng (2020) finds that inflation mitigates income inequality in that it lowers real interest rate and thereby in turn mitigates the contribution of wealth heterogeneity, which again is the sole source of income inequality. The calibrated economy in Zheng *et al.* (2020) also suggests a negative nexus between inflation and income inequality. Because these studies all regard the wealth heterogeneity as the only source of income inequality and abstract away from skill heterogeneity, a lower (higher) contribution of interest income is necessarily related with a lower (higher) degree of income inequality. The present study breaks this relation by adding skill heterogeneity and thus complements the above interesting studies as it enables us to consider both interest income inequality and labor income inequality.[8] More importantly, our paper thereby generates mixed results (i.e., negative, positive and U-shaped) on inflation and income inequality that help to explain the empirical inconsistency.

This study also relates to a fast-growing literature on investigating income disparity in R&D-based growth models, such as Chou and Talmain (1996), Foellmi and Zweimüller (2006), García-Peñalosa and Wen (2008), Chu and Cozzi (2018), Jones and Kim (2018) and Aghion *et al.* (2019). These studies mainly focus on the relation between innovation and income (wealth) inequality. Our study contributes to this literature by decomposing income inequality into interest and labor income inequalities, and examining the effects of monetary policy on income inequalities jointly through these two sources.

The rest of this study proceeds as follows. The basic model is spelled out in Section 2. Section 3 characterizes the wealth distribution and the income distribution and investigates the effect of monetary policy on income inequality. Section 4 provides a quantitative analysis. The final section concludes.

2 The model

In this section, we extend the version of the quality-ladder growth model in Acemoglu (2009) (Chapter 14), which originates from Grossman and Helpman (1991), in the following aspects.

[7] Afonso and Sequeira (2022) also investigate the effects of inflation on specialization, growth and wage inequality. However, they do not study the relation between inflation and inequality, which is the focus of this paper.

[8] When considering elastic labor supply, this line of research can also produce an unequal distribution of labor income. However, the labor income distribution in previous studies is essentially determined by the wealth distribution because the amount of working time by each household is related to her asset endowments. By adding the skill dimension of heterogeneity, our model breaks this strong link between labor income heterogeneity and wealth heterogeneity.

First, we introduce household heterogeneity by assuming that households possess different levels of initial wealth endowments as in García-Peñalosa and Turnovsky (2006) and skill endowments as in García-Peñalosa and Turnovsky (2015). Second, we model money demand via a CIA constraint on R&D investment as in Chu and Cozzi (2014). Moreover, the nominal interest rate serves as the monetary policy instrument and the effects of monetary policy are examined by considering the implications of altering the rate of nominal interest on economic growth and income inequality, respectively.

2.1 Households

The economy is populated by a unit continuum of households indexed by $s \in [0,1]$. Households share the same preference over consumption $c_t(s)$, and the lifetime utility function for each household s is

$$U(s) = \int_0^\infty e^{-\rho t} \ln c_t(s) dt, \tag{1}$$

where $\rho > 0$ represents the discount rate.

The budget constraint for household s (expressed in units of final good) is

$$\dot{a}_t(s) + \dot{m}_t(s) = r_t a_t(s) + i_t b_t(s) - \pi_t m_t(s) + w_t(s) - c_t(s) + \tau_t, \tag{2}$$

where $a_t(s)$ is the real value of financial assets owned by household s and r_t is the real interest rate. $m_t(s)$ is the real value of money balance by household s and π_t is the inflation rate reflecting the opportunity cost of holding money. $b_t(s)$ is the amount of cash borrowed from household s by entrepreneurs for R&D, and the rate of return on $b_t(s)$ is i_t. The household $s \in [0,1]$ is endowed with $h(s)$ units of skills, which are assumed to be stationary over time. Each household supplies one unit of labor inelastically, providing $1 \cdot h(s)$ units of skill augmented labor, or effective labor, and w_t is the real wage rate. Then $h \equiv \int_0^1 h(s)ds$ is the aggregate supply of effective labor.[9] Moreover, each household receives an identical amount τ_t of lump-sum transfer from the government. Another constraint faced by each household is the CIA constraint such that[10]

$$b_t(s) \leq m_t(s). \tag{3}$$

Households maximize lifetime utility subject to the budget constraint (2) and the CIA con-

[9] To ensure the tractability of this model, we also assume that households do not make efforts on accumulating their own skills. See Turnovsky and Mitra (2013) for a more sophisticated model that allows both endogenous physical and human capital accumulation.

[10] Owing to the empirical consistency with R&D-cash flow sensitivity, we model CIA constraint on R&D. However, we do not consider CIA constraint on consumption because the CIA constraint on consumption indicates that the distribution of consumption across household would be identical to that of money holdings. This result would contradict with the empirical facts documented by Ragot (2014) who uses Italy and the US data to find that the distribution of money (MI) across households coincides with the distribution of other financial assets rather than that of the consumption expenditure.

straint (3). From standard dynamic optimization,[11] using the optimality condition for real money balance $m_t(s)$ and $b_t(s)$, we can derive a no-arbitrage condition between financial assets and money given by $i_t = r_t + \pi_t$; therefore, i_t is also the nominal interest rate. We then derive the familiar Euler equation

$$\dot{c}_t(s)/c_t(s) = r_t - \rho. \tag{4}$$

This equation implies that the growth rates of real consumption are identical across households such that $\dot{c}_t(s)/c_t(s) = \dot{c}_t/c_t$, where $c_t \equiv \int_0^1 c_t(s)ds$ denotes the aggregate consumption of final good by all households.

2.2 Final good

There is a unique final good in the economy that is produced by a mass of perfectly competitive firms. The production factors are efficient labor h and a continuum of differentiated intermediate goods. The aggregate production function takes the following form

$$y_t = \frac{h^{1-\alpha}}{\alpha} \int_0^1 q_t(\epsilon) x_t(\epsilon|q)^\alpha d\epsilon, \quad \alpha \in (0,1) \tag{5}$$

where $x_t(\epsilon|q)$ is the quantity of intermediate good in industry $\epsilon \in [0,1]$, whose quality at time t is $q_t(\epsilon)$. The quality evolves as follows

$$q_t(\epsilon) = q_0(\epsilon)\lambda^{n_t(\epsilon)}, \tag{6}$$

where q_0 is the quality level at time 0, $\lambda > 1$ measures the quality step size of each innovation, and $n_t(\epsilon)$ denotes the number of innovations on this product line between time 0 and t. From profit maximization, we obtain the conditional demand functions for h and $x_t(\epsilon|q)$, respectively,

$$h = (1-\alpha)y_t/w_t. \tag{7}$$

and

$$x_t(\epsilon|q) = \left(\frac{q_t(\epsilon)}{p_t(\epsilon|q)}\right)^{\frac{1}{1-\alpha}} h \tag{8}$$

where $p_t(\epsilon|q)$ is the price of $x_t(\epsilon|q)$.

2.3 Intermediate goods

The differentiated intermediate goods in industry ϵ are produced by a monopolistic leader who holds a patent on the latest innovation. The leader's products would not be replaced until a new entrant who has a more advanced innovation comes into the market. The marginal cost of

[11] See Appendix A.1 for detailed derivations.

producing a unit of intermediate good is $\eta q_t(\epsilon)$ units of final good, where $\eta \in (0,1)$. The fact that the marginal cost is proportional to the quality of the intermediate good means that producing higher-quality products should be more expensive. Thus, the ϵ-th intermediate good producer maximizes her/his profits $\Pi_t(\epsilon) = [p_t(\epsilon|q) - \eta q_t(\epsilon)]x_t(\epsilon|q)$, subject to the demand function in (8). Solving this profit-maximizing problem gives rise to the monopolistic price[12]

$$p_t(\epsilon|q) = q_t(\epsilon). \tag{9}$$

where we have normalized $\eta = \alpha$ without loss of generality.[13] Combining (8) and (9) yields

$$x_t(\epsilon|q) = h. \tag{10}$$

Consequently, the flow profits of a firm with the latest production technology $q_t(\epsilon)$ can be computed as

$$\Pi_t(\epsilon|q) = (1-\alpha)q_t(\epsilon)h. \tag{11}$$

2.4 Innovations and R&D

Denote by $v_t(\epsilon|q)$ the real value of a firm who holds the most recent innovation in industry ϵ. Accordingly, the Hamilton-Jacobi-Bellman (HJB) equation for $v_t(\epsilon|q)$ is

$$r_t v_t(\epsilon|q) = \Pi_t(\epsilon|q) + \dot{v}_t(\epsilon|q) - \mu_t(\epsilon|q)v_t(\epsilon|q), \tag{12}$$

which is also the no-arbitrage condition for the value of the asset (in the form of a patented innovation). Equation (12) says that the return on this asset $r_t v_t(\epsilon|q)$ equals to the sum of the flow profits $\Pi_t(\epsilon|q)$, the potential capital gain $\dot{v}_t(\epsilon|q)$, and the loss $\mu_t(\epsilon|q)v_t(\epsilon|q)$ due to creative destruction, where $\mu_t(\epsilon|q)$ is the rate at which new innovations occur in sector ϵ at time t.

We now specify the technology for producing new vintages of intermediate goods, based on the lab-equipment specification. If a firm spends $z_t(\epsilon|q)$ units of final good for research in product line ϵ when quality is at level q, it then generates a flow rate

$$\mu_t(\epsilon|q) = \frac{\varphi z_t(\epsilon|q)}{h q_t(\epsilon)} \tag{13}$$

of innovation, where $\varphi > 0$ is a productivity parameter. Innovation advances the know-how of production of this intermediate good to a new rung of the quality ladder, creating a superior product with quality $\lambda q_t(\epsilon)$. Equation (13) implies that the probability of the next successful

[12]As in Howitt (1999), we hereby make an assumption that once the incumbent stops production and leaves the market, she cannot threaten to reenter. Therefore, the local monopolist can charge the unconstrained monopolistic price without worrying about competition from earlier vintages of the product.

[13]We consider a more general case with $\eta \neq \alpha$ in the Appendix B and find that the qualitative results remains unchanged.

innovation is increasing in R&D expenditures $z_t(\epsilon|q)$ whereas decreasing in quality $q_t(\epsilon)$, capturing the insight that research on more advanced products becomes more difficult, so one unit of R&D spending is proportionately less effective when applied to a more sophisticated product. Moreover, to neutralize the scale effect in this model, we follow Barro and Sala-i Martin (2004) and Annicchiarico *et al.* (2022) by assuming that the flow rate of innovation depends on R&D expenditures per unit of effective labor.[14]

We assume that there is free-entry into research and that R&D investment is always positive. In addition, to capture firms' cash requirement on innovative activities, following the existing literature such as Chu and Cozzi (2014) and Chu *et al.* (2015), we assume that each firm needs to borrow money to facilitate its R&D expenditures subject to the nominal interest rate i_t. To parameterize the strength of this CIA constraint, we make another assumption that a fraction $\kappa \in [0,1]$ of R&D investment requires the borrowing of money from households such that $b_t(\epsilon) = \kappa z_t(\epsilon|q)$. Therefore, the total cost of R&D is $(1+\kappa i_t)z_t(\epsilon|q)$. Following Acemoglu (2009), free entry implies that the expected profit on R&D spending $z_t(\epsilon|q)$ in line ϵ that has quality $q\lambda^{-1}$ at time t must be zero such that $\mu_t(\epsilon|q\lambda^{-1})v_t(\epsilon|q) - (1+\kappa i_t)z_t(\epsilon|q\lambda^{-1}) = 0$. Combining this equation with (13) yields

$$v_t(\epsilon|q) = \frac{hq_t(\epsilon)(1+\kappa i_t)}{\varphi\lambda}. \tag{14}$$

2.5 Monetary authority

We model the monetary sector in line with Chu and Cozzi (2014) and Gil and Iglésias (2020). Denote the aggregate nominal money supply by M_t and its growth rate by $\varepsilon_t \equiv \dot{M}_t/M_t$, respectively. Accordingly, the aggregate real money balance is given by $m_t \equiv \int_0^1 m_t(s)ds = M_t/P_t$, where P_t is the nominal price of final good. Then, given an exogenously chosen i_t by monetary authority, the inflation rate is endogenously determined according to the Fisher equation, $i_t = r_t + \pi_t$. Given π_t, the growth rate of nominal money supply is endogenously determined according to $\varepsilon_t = \dot{m}_t/m_t + \pi_t$.

We assume that the monetary authority returns all its seigniorage revenue to households and the government runs a balanced budget such that $\tau_t = \int_0^1 \dot{m}_t(s)ds + \pi_t \int_0^1 m_t(s)ds = \varepsilon_t m_t$, where $\varepsilon_t = \dot{m}_t/m_t + \pi_t$ has been applied. The right-hand side of this equation is the total seigniorage revenue, and the left-hand side is the government's transfer, implying that the government rebates the seigniorage revenue back to each household in a uniform lump-sum fashion.[15]

[14] See Barro and Sala-i Martin (2004) and Annicchiarico *et al.* (2022) for a more detailed discussion for this setup.

[15] Alternatively, as stressed in Chu and Cozzi (2014), one can also consider the growth rate of money supply ε_t as the policy instrument. In this case, combining $\pi_t = \varepsilon_t - \dot{m}_t/m_t$ with the Fisher equation (i.e., $i_t = \pi_t + r_t$), alone with the Euler equation, yields the one-to-one relation between the nominal interest rate and the nominal money supply in the balanced growth path equilibrium $i_t = r_t + \pi_t = (\rho + g_t) + (\varepsilon_t - g_t) = \varepsilon_t + \rho$, where we have applied the condition that on the balanced growth path the aggregate consumption and real money balance grow at the same rate of $r_t - \rho$ according to the Euler equation, which will be shown in Lemma 1. Moreover, the long-run equilibrium relationships also imply that our analysis on how the nominal interest rate relates to innovation, economic growth, and income

2.6 Aggregation

Substituting (10) into (5) yields the total output

$$y_t = Q_t h/\alpha, \tag{15}$$

where

$$Q_t \equiv \int_0^1 q_t(\epsilon)d\epsilon \tag{16}$$

is defined as the average total quality of intermediate goods. Note that although the qualities $q_t(\epsilon)$ are stochastic, their average Q_t is deterministic for the law of larger numbers. Let $x_t = \int_0^1 \eta q_t(\epsilon)x_t(\epsilon|q)d\epsilon$ denote the aggregate expenditure on final good used to produce intermediate goods. Then using (9) and (10), together with the normalized condition $\eta = \alpha$, we have

$$x_t = \alpha Q_t h. \tag{17}$$

Substituting (15) into (7), we also can derive the equilibrium real wage rate

$$w_t = \left(\frac{1-\alpha}{\alpha}\right) Q_t. \tag{18}$$

Moreover, from (11) and (16), we derive the total profits of the intermediate-goods sector:

$$\Pi_t \equiv \int_0^1 \Pi_t(\epsilon|q)d\epsilon = (1-\alpha)Q_t h. \tag{19}$$

Finally, denote by v_t the market aggregate value of firms in the intermediate-goods sector, which is

$$v_t = \int_0^1 v_t(\epsilon|q)d\epsilon = \frac{(1+\kappa i_t)Q_t h}{\varphi\lambda}, \tag{20}$$

where again (16) has been used.

2.7 Decentralized equilibrium

We now define the decentralized equilibrium in the economy. An equilibrium is represented as time paths of effective labor, consumption levels, aggregate spending on intermediate goods, and aggregate R&D expenditure, $[h, c_t, x_t, z_t]_{t=0}^{\infty}$, where $z_t \equiv \int_0^1 z_t(\epsilon|q)d\epsilon$ denotes the aggregate R&D spending; stochastic paths of prices and quantities for intermediate goods that have highest quality in their lines at that point, $[p_t(\epsilon|q), x_t(\epsilon|q)]_{\epsilon\in[0,1],t=0}^{\infty}$; and time paths of aggregate quality, $[Q_t]_{t=0}^{\infty}$, real and nominal interest rates, $[r_t, i_t]_{t=0}^{\infty}$, real wage rates $[w_t]_{t=0}^{\infty}$, real bond and money holdings, $[b_t, m_t]_{t=0}^{\infty}$, and value functions, $[v_t(\epsilon|q)]_{\epsilon\in[0,1],t=0}^{\infty}$ such that

inequality, also applies to the counterpart on how inflation relates to those variables.

1. heterogeneous households $s \in [0,1]$ maximize their utility taking $\{r_t, i_t, w_t, \tau_t\}$ as given;
2. competitive final-good firms produce y_t to maximize profits taking $\{w_t, p_t(\epsilon|q)\}$ as given;
3. monopolistic intermediate-goods firms produce $x_t(\epsilon|q)$ to maximize profits taking $\{P_t\}$ as given;
4. competitive R&D firms choose $\{z_t(\epsilon|q)\}$ to maximize their profits taking $\{r_t, i_t, P_t\}$ as given;
5. the market-clearing condition for labor holds;
6. the market-clearing condition for final good holds such that

$$c_t + x_t + z_t = y_t; \tag{21}$$

7. the market-clearing condition for financial assets holds such that

$$a_t \equiv \int_0^1 a_t(s)ds = v_t; \tag{22}$$

8. the innovating firms finance their R&D spending through borrowing

$$b_t \equiv \int_0^1 b_t(s)ds = \kappa \int_0^1 z_t(\epsilon|q)d\epsilon = \kappa z_t; \tag{23}$$

9. and monetary authority balances its budget such that $\tau_t = \varepsilon_t m_t = (i_t - \rho)m_t$.

We show in the following lemma that the dynamic property of the aggregate economy is similar to Chu and Cozzi (2014), which is based on the "knowledge-driven" setting.

Lemma 1. *Given a constant nominal interest rate i, the economy immediately jumps to a unique and saddle-point stable balanced growth path along which variables $\{y_t, c_t, x_t, z_t, Q_t, v_t, b_t, w_t, m_t\}$ grow at the same and constant rate.*

Proof. See Appendix A.2. □

From Lemma 1, given a stationary time path of nominal interest rate, we can derive the steady-state levels of several variables along the BGP as in the following Lemma.

Lemma 2. *The steady-state aggregate R&D spending is*

$$z_t = \frac{\mu Q_t h}{\varphi}. \tag{24}$$

The steady-state arrival rate of innovation and growth rate of aggregate quality are given by, respectively,

$$\mu = \frac{\varphi(1-\alpha)}{1+\kappa i} - \frac{\rho}{\lambda}. \tag{25}$$

The steady-state growth rate of aggregate quality is obtained by substituting (25) *into* (A.19) *such that*

$$g = \frac{\varphi(1-\alpha)(\lambda-1)}{1+\kappa i} - \frac{\rho(\lambda-1)}{\lambda}. \tag{26}$$

Proof. See Appendix A.3. □

It can be seen that the growth rate of aggregate quality in (26) is independent of the size of the effective labor supply. The scale effect is therefore removed. Moreover, equations (25) and (26) imply that a higher nominal interest rate reduces the steady-state innovation arrival rate and thereby causes a decline in the steady-state economic growth rate. Intuitively, raising the nominal interest rate increases the marginal cost of R&D and therefore discourages innovation incentives. As a result, the resources devoted to the research sector decreases, and the aggregate flow rate of innovation declines in response. This result is unsurprising and is consistent with the theoretical prediction by Chu and Cozzi (2014) and Huang *et al.* (2017).[16] Formally, we establish the following proposition.

Proposition 1. *The growth rates of output, aggregate consumption, and technology are all decreasing in the nominal interest rate.*

Proof. Proven in the text. □

3 Monetary Policy and Income Inequality

In this section, we first follow García-Peñalosa and Turnovsky (2015) to characterize the skill and wealth distributions (including the financial assets and bonds issued by firms) and show that both distributions are stationary along the balanced growth path. Then we derive the income distribution and analyze the effect of monetary policy on income inequality.

3.1 Skill distribution and wealth distribution

Denote by $\theta_{h,0}(s) \equiv h(s)/h$ the relative skill of household s at time 0. Since in our model all households' skill levels are exogenously given, the relative skill by household s is time invariant such that $\theta_{h,t}(s) = \theta_{h,0}(s)$. Therefore, the heterogeneity of relative skills across households can be described by a stationary distribution with a mean of one and a constant standard deviation $\sigma_h > 0$.

[16]It also receives supports from the empirical findings in Vaona (2012) and Barro (2013).

We next characterize the dynamic property of wealth distribution. Denote by $d_t(s) \equiv a_t(s) + b_t(s)$ and $d_t \equiv a_t + b_t$ household s' wealth and average/aggregate wealth of all households at time t, respectively. In addition, define by $\theta_{d,t}(s) \equiv d_t(s)/d_t$ the relative wealth of household s at time t and $\theta_{d,0}(s) \equiv d_0(s)/d_0$ the initial relative wealth of household s, respectively. At any point of time the relative wealth has mean one, and its dispersion across households is given by the standard deviation, $\sigma_{d,t}$, and its initial given dispersion is σ_d.

Since household s exhausts all her cash in equilibrium such that $b_t(s) = m_t(s)$, her budget constraint in (2) can be rewritten as $\dot{d}_t(s) = r_t d_t(s) + w_t h(s) - c_t(s) + \tau_t$. Aggregating it for all s, we obtain

$$\dot{d}_t = r_t d_t + w_t h - c_t + \tau_t. \tag{27}$$

We thus derive the motion $\theta_{d,t}(s)$ such that

$$\dot{\theta}_{d,t}(s) = \frac{c_t - \tau_t - w_t h}{d_t}\theta_{d,t}(s) - \frac{c_t\theta_{c,t}(s) - \tau_t - w_t h\theta_{h,0}(s)}{d_t}, \tag{28}$$

where $\theta_{c,t}(s) \equiv c_t(s)/c_t$ is the relative consumption level by household s at time t. Log-differentiating $\theta_{c,t}(s)$ with respect to time yields

$$\frac{\dot{\theta}_{c,t}(s)}{\theta_{c,t}(s)} = \frac{\dot{c}_t(s)}{c_t(s)} - \frac{\dot{c}_t}{c_t} = 0, \tag{29}$$

according to the Euler equation (4). Equation (29) then implies $\theta_{c,t}(s) = \theta_{c,0}(s)$ for all t. We show in Appendix A.4 that the coefficient on $\theta_{d,t}(s)$ in (28) equals to $\rho > 0$. Since $\theta_{d,t}(s)$ is a state variable, the only solution consistent with long-run stability is $\dot{\theta}_{d,t}(s) = 0$ for all t. It is achieved by setting the relative consumption level $\theta_{c,t}(s)$, a control variable, to its steady state value $\theta_{c,0}(s)$. We thus obtain a convenient property of the wealth distribution as illustrated in the following lemma.

Lemma 3. *For each household s, the relative skill and wealth are constant over time and exogenously determined at time 0 such that $\theta_{h,t}(s) = \theta_{h,0}(s)$ and $\theta_{d,t}(s) = \theta_{d,0}(s)$ for all time $t > 0$, respectively.*

Proof. See Appendix A.4. □

Although the distributions of skill and wealth are neutral to monetary policy, as will be shown in the next section, monetary policy affects the relative contribution of the two types of heterogeneity through the ratio of interest income to labor income, causing income distribution to be endogenously determined.

3.2 Income Distribution

The amount of before-transfer income earned by each household s is the sum of the interest income and labor income such that $I_t(s) = r_t d_t(s) + w_t h(s)$. The total income earned by all households is therefore $I_t = r_t d_t + w_t h$. Combining both equations yields the relative income earned by household s

$$\theta_{I,t}(s) \equiv \frac{I_t(s)}{I_t} = \frac{r_t d_t \theta_{d,0}(s) + w_t h \theta_{h,0}(s)}{r_t d_t + w_t h}, \tag{30}$$

where the second equality applies $\theta_{d,t}(s) = \theta_{d,0}(s)$ from Lemma 3. Because both the wealth and skill distribution functions have a mean of one, the distribution of relative income at time t also has mean one and the following variance

$$\begin{aligned}
\sigma_{I,t}^2 \equiv \int_0^1 [\theta_{I,t}(s) - 1]^2 ds &= \left(\frac{r_t d_t}{r_t d_t + w_t h}\right)^2 \sigma_d^2 + \left(\frac{w_t h}{r_t d_t + w_t h}\right)^2 \sigma_h^2 \\
&= \left(\frac{r_t d_t / w_t h}{r_t d_t / w_t h + 1}\right)^2 \sigma_d^2 + \left(\frac{1}{r_t d_t / w_t h + 1}\right)^2 \sigma_h^2 \\
&= \left(\frac{\Phi_t}{1 + \Phi_t}\right)^2 \sigma_d^2 + \left(\frac{1}{1 + \Phi_t}\right)^2 \sigma_h^2,
\end{aligned} \tag{31}$$

where $\Phi_t \equiv r_t d_t / w_t h$ is the ratio of interest income to labor income. In the above equation, we follow Jin (2009) by assuming a zero covariance (correlation) $\sigma_{d,h} = 0$, and will discuss the case of non-zero covariance in the quantitative part.

Equation (31) shows that the degree of income inequality, measured by the variance of income distribution $\sigma_{I,t}^2$, can be decomposed into the variances of wealth distribution σ_d^2 and of skill distribution σ_h^2. Since both distributions are unaffected by the nominal interest rate according to Lemma 3, the impact of i on $\sigma_{I,t}^2$ is boiled down to the influence of i on the relative contribution of wealth heterogeneity to skill heterogeneity, which is governed by the ratio of interest income to labor income, Φ_t. In literature such as Chu (2010) and Chu *et al.* (2019), in which households' different amounts of labor income are determined by their unequal levels of asset/wealth endowments, instead of skills, the degree of income inequality is monotonically increasing in Φ_t, because a higher ratio causes the income distribution to be more determined by wealth heterogeneity. Nevertheless, introducing labor income heterogeneity in our model alters this relation. In particular, a rise in Φ_t increases the degree of income inequality only if the relative variance of wealth distribution to skill distribution is sufficiently large (i.e., smaller σ_h^2 / σ_d^2), and decreases it otherwise. The following lemma summarizes these results.

Lemma 4. *The degree of income inequality $\sigma_{I,t}^2$ is increasing in the ratio of interest income to labor income Φ_t if $\Phi_t > \sigma_h^2 / \sigma_d^2$, and decreasing in Φ_t if $\Phi_t < \sigma_h^2 / \sigma_d^2$.*

Proof. Differentiating (31) with respect to Φ_t shows that $\partial \sigma_{I,t}^2 / \partial \Phi_t \gtrless 0 \Leftrightarrow \Phi_t \sigma_d^2 \gtrless \sigma_h^2$. □

3.3 Effect of monetary policy on income inequality

Before analyzing the influence of the nominal interest rate on income inequality, we first need to explore how a rise in i affects the ratio of interest income to labor income. Recall that the total wealth in the economy is $d_t = a_t + b_t$. Using (20) and (22), the amount of financial assets a_t is derived such that

$$a_t = v_t = \frac{Q_t h(1+\kappa i)}{\varphi\lambda}, \tag{32}$$

Using (23), (24) and (25), we derive the amount of borrowing b_t

$$b_t = \kappa z_t = \frac{\kappa Q_t h}{\varphi}\mu = \frac{\kappa Q_t h}{\varphi}\left(\frac{\varphi(1-\alpha)}{1+\kappa i} - \frac{\rho}{\lambda}\right). \tag{33}$$

Combining (32) and (33), alone with (18), we obtain the ratio of wealth to labor income given by

$$\frac{d_t}{w_t h} = \underbrace{\frac{\frac{Q_t h(1+\kappa i)}{\varphi\lambda}}{(1-\alpha)Q_t h/\alpha}}_{a_t/w_t h} + \underbrace{\frac{\frac{\kappa Q_t h}{\varphi}\left(\frac{\varphi(1-\alpha)}{1+\kappa i} - \frac{\rho}{\lambda}\right)}{(1-\alpha)Q_t h/\alpha}}_{b_t/w_t h} = \frac{\alpha(1+\kappa i-\kappa\rho)}{\lambda\varphi(1-\alpha)} + \frac{\alpha\kappa}{1+\kappa i}, \tag{34}$$

which is stationary in equilibrium. Furthermore, together with the fact that $r = \rho + g$ in (4) is also stationary, we thus can infer that $\Phi_t = \Phi$ is time invariant

$$\Phi = \left\{\frac{\varphi(1-\alpha)(\lambda-1)}{1+\kappa i} + \frac{\rho}{\lambda}\right\} \cdot \left\{\frac{\alpha(1+\kappa i-\kappa\rho)}{\lambda\varphi(1-\alpha)} + \frac{\alpha\kappa}{1+\kappa i}\right\}. \tag{35}$$

where we have applied (26). Moreover, the variance of income distribution in (31) is therefore stationary, $\sigma^2_{I,t} = \sigma^2_I$.

We can see that a change in i affects Φ through two channels: by affecting the real interest rate r, and the ratio of wealth to labor income $d_t/w_t h$. First, according to Proposition 1, a higher i reduces the economic growth rate, and thus lowers the real interest rate according to the Euler equation (4). This effect, referred to as the *interest-rate effect* and identified by Chu and Cozzi (2018), tends to cause the ratio of interest income to labor income to decline. Second, increasing i has two additional effects: (i) raises the ratio of financial assets to labor income $a_t/w_t h$, because it increases the asset value a_t by driving up the unit cost of R&D via the free-entry condition in (14). This is known as the *asset-value effect*; (ii) creates a negative effect on the ratio of bond value to labor income $b_t/w_t h$, as increasing i reduces the R&D spending and thereby the demand for money (bonds) for R&D activities. This is known as the *bond-value effect*. The latter two effects combined are called the *wealth-value effect*. When the intermediate-goods share of output α is sufficiently large and for a wide range of parameters, the positive *asset-value* effect tends to dominate the negative *interest-rate* effect and *bond-value* effect, causing the impact of i on Φ to be monotonically increasing. Yet, when α is sufficiently small, the overall effect of i on Φ is U-shaped; that is, a rise in i from zero would first trigger a decline in the ratio of interest income to labor income, followed by an increase as i gets larger and exceeds the critical value where the ratio reaches the lowest. Lemma 5 summarizes these results.

Lemma 5. *The ratio of interest income to labor income can be a monotonically increasing function of nominal interest rate i if $\alpha \in [\alpha_T, 1-\rho/\lambda\varphi)$ and $\rho < \min(1/\lambda, \lambda\varphi(1-\alpha))$, and a U-shaped function if $\alpha \in [0, \alpha_T)$ and $\rho < \min(1/\lambda, \lambda\varphi(1-\alpha), \varphi\lambda\kappa[\rho(2-\lambda)(1-\alpha) - 2\varphi\lambda(\lambda-1)(1-\alpha)^2])$.*

Proof. See Appendix A.5. □

Having established Lemmas 4 and 5, we now turn to investigate the impact of a rise in nominal interest rate on income inequality. Differentiating σ_I^2 in (31) with respect to i shows

$$\frac{\partial \sigma_I^2}{\partial i} \gtreqless 0 \Leftrightarrow \left(\Phi \sigma_d^2 - \sigma_h^2\right) \cdot \frac{\partial \Phi}{\partial i} \gtreqless 0. \tag{36}$$

It is straightforward that the relation between the nominal interest rate and income inequality is jointly determined by two terms $\Phi\sigma_d^2 \gtreqless \sigma_h^2$ and $\partial\Phi/\partial i \gtreqless 0$.

In the following analysis, we focus on the case in which the intermediate-goods share of output α is sufficiently large, implying that the ratio of interest income to labor income is increasing in the nominal interest rate, i.e., $\partial\Phi/\partial i > 0$ for all $i > 0$ from Lemma 5.[17] In this circumstance, if the wealth heterogeneity is larger than the skill heterogeneity in the zero-nominal-interest-rate environment such that $\Phi_{i=0} > \sigma_h^2/\sigma_d^2$, Lemma 4 then implies that the income inequality σ_I^2 is increasing in the nominal interest rate at $i = 0$. Given that Φ monotonically increases in i, the condition $\Phi_{i>0} > \sigma_h^2/\sigma_d^2$ continues to hold as i rises from 0 to $\hat{i}$.[18] The intuition behind is that the positive *asset-value* effect dominates both negative *interest-rate* and *bond-value* effects such that a rise in i increases Φ, thus driving up the relative contribution of wealth heterogeneity on income inequality in equation (31).

By contrast, if the wealth heterogeneity is smaller than the skill heterogeneity in the zero-nominal-interest-rate environment such that $\Phi_{i=0} < \sigma_h^2/\sigma_d^2$, then increasing the nominal interest rate leads to a decline in the income inequality at $i = 0$. Intuitively, it increases the relative contribution of σ_d^2 which has a relatively small dispersion but decreases the weight of σ_h^2 which has a relatively large dispersion. As long as the condition $\Phi_{0<i<\hat{i}} < \sigma_h^2/\sigma_d^2$ continues to hold as i increases and approaches $\hat{i}$, a higher i always reduces income inequality. However, if the sign of condition $\Phi_{0<i<\hat{i}} < \sigma_h^2/\sigma_d^2$ is reversed as i rises, then raising the nominal interest rate turns to enlarge income inequality. Therefore, there may exist a threshold value $\tilde{i}$ below which $\Phi_{0<i<\tilde{i}} < \sigma_h^2/\sigma_d^2$ and above which $\Phi_{\tilde{i}<i<\hat{i}} > \sigma_h^2/\sigma_d^2$. In this circumstance, a rise in i would first induce a decrease in income inequality and an increase in it afterwards; that is, a U-shaped impact on income inequality. The following proposition summarizes these results.

[17] Due to the mathematical complexity, we resort to numerical experiments on simulating the relation between nominal interest rate and income inequality under an insufficiently large α, as shown below in Section 4.3.

[18] Throughout the analysis, we focus on the circumstances of non-negative innovation arrival rate and economic growth rate. It thus implies an upper bound of nominal interest rate $\hat{i} \leq +\infty$ that guarantees $\mu \geq 0$ in (25). Expression for $\hat{i}$ is provided in Appendix A.5.

Proposition 2. *Given a zero covariance between wealth and skill heterogeneities and a sufficiently large intermediate-goods share of output* α*, the degree of income inequality* σ_I^2 *can be a monotonically increasing function of the nominal interest rate* i *if* $\Phi_{i=0} > \sigma_h^2/\sigma_d^2$*, and a U-shaped or monotonically decreasing function of* i *if* $\Phi_{i=0} < \sigma_h^2/\sigma_d^2$*.*

Proof. Proven in text. □

4 Quantitative analysis

We first calibrate the model to the data of the US economy in Section 4.1, and then simulate the impacts of a rise in inflation on economic growth rate and income inequality in Section 4.2. Section 4.3 perform a sensitivity analysis on the intermediate-goods share of output α, and Section 4.4 considers more general cases with a non-zero covariance between the wealth and skill heterogeneities.

4.1 Calibration

The model features the following structural parameters $\{\rho, \alpha, \lambda, \varphi, \kappa, i\}$. We follow Acemoglu and Akcigit (2012) in choosing a conventional value of 0.05 for the discount rate ρ. As for the intermediate-goods share of output α, we calibrate it by matching the labor share of GDP. As documented in Elsby *et al.* (2013),[19] labor's share of income in the United States has trended downward and fallen to around 58% since the beginning of the new century. We therefore set the benchmark value of labor's share of income to 58%, which corresponds to $w_t h/(y_t - x_t)$ in our model. Using (15), (17) and (18), we have $1/(1+\alpha) = 0.58$, implying $\alpha = 0.724$. As for the quality step size of innovation, λ, we calibrate it by matching the growth rate of TFP and the innovation arrival rate. First, the growth rate of TFP in the US is around 0.6% during the period $2000 - 2018$ (according to data retrieved from FRED, Federal Reserve Bank of St. Louis). Second, we set the benchmark innovation arrival rate to $\mu = 6\%$.[20] Therefore, the benchmark quality step size of innovation is obtained such that $\lambda = 1 + g/\mu = 1.1$. This calibrated quality step size is also consistent with the estimate from Akcigit and Kerr (2018), who find a radical innovation step size of 1.112 and an incremental one of 1.051.

As for the remaining parameter κ, φ, and i, in addition to matching the above benchmark innovation arrival rate, we adopt another two indicators to pin down their values; that is, the average inflation rate and the average ratio of wealth to labor income. First, we pin down the

[19] See also vom Lehn (2018) and Bergholt *et al.* (2022) for extended discussion on the declining labor share in the U.S.

[20] The existing literature has considered different values for the arrival rate of innovations. For instance, Caballero and Jaffe (2002) and Laitner and Stolyarov (2013) estimate a mean rate of creative destruction around 4% and 3.5%, and Lanjouw (1998) reports the probability of obsolescence to be situated within 7% to 12%. We consider an intermediate value of 6% within this range.

benchmark nominal interest rate by targeting the average inflation rate in the US during $2000-2019$, which is 2.1% according to FRED. Thus, the implied nominal interest rate is $i = r + \pi = \rho + g + \pi = 7.7\%$. Second, according to the 2019 Survey of Consumer Finances, during the period $2001-2019$, the average mean family income and the average mean family net worth are 100.3 and 647.3 thousands of 2019 dollars, respectively. This corresponds to the ratio $d_t/w_t h = 6.45$ in (34) in our model. We use both (25) and (34) to calibrate κ and φ. Table 1 summarizes our benchmark parameter values and targeted empirical moments.

Finally, to simulate the effects of inflation rate/nominal interest rate on income inequality, we need to determine the initial relative variance of wealth and skill distributions, σ_d^2 and σ_h^2.[21] In the literature, empirical studies have reported different values of variances of wealth and income distributions mainly because of the different datasets explored. For example, based on a sample of historical data $(1981-1987)$ from the Panel Study of Income and Dynamics (PSID),[22] Shea (1995) reported standard deviations for income distribution 10200 and wealth distribution 24705, respectively. Therefore, from (31) and given above calibrated parameters, we can compute the relative variance of skill to wealth distribution as $\sigma_h^2/\sigma_d^2 = 0.185$. Moreover, Blau (1999) adopted another sample of the National Longitudinal Survey of Youth (NLSY) and found the standard deviations of total family income and non-labor income distribution are 0.74 and 0.66, respectively.[23] The relative variance of skill to wealth distribution in this case is $\sigma_h^2/\sigma_d^2 = 2.199$. We thus consider an intermediate value of $\sigma_h^2/\sigma_d^2 = 0.365$ within this range as the benchmark.[24]

4.2 Benchmark simulation

In this calibrated economy, we find that the economic growth rate is a decreasing function of the inflation rate, as depicted in Figure 1a. This result confirms the model prediction in Proposition 1.[25] Intuitively, a higher inflation rate (a higher nominal interest rate) increases the firms' borrowing cost for R&D activities, and thus depresses innovation and economic growth (technology/output growth). In particular, when raising the inflation rate from $\pi = -5.64\%$ (where the nominal interest rate is zero) to $\pi = 15\%$ (i.e., $i = 20.54\%$), the economic growth rate

[21] We do not use Gini coefficient as inequality measures because equation (31) would request a decomposition of Gini coefficient into various income sources. The decomposition method has been extensively studied and is rather nontrivial. Moreover, there is a lack of critical data for decomposition, such as the "rank correlation" data that is needed for constructing the Pseudo Gini coefficient. See Shorrocks (1982) and Lerman and Yitzhaki (1985) for an in-depth analysis.

[22] Since the above targeted moments mainly cover the period after 2000, it is arguably plausible to select the datasets before 2000 as the initial period.

[23] The sample data here is in 10,000 1979 dollars, while the sample data used in Shea (1995) is in 1982 dollars.

[24] We have also considered other values of the relative variance of skill to wealth distribution in this range, and found that the positive and U-shaped relations between inflation and income inequality documented below still hold. For saving space, we do not report these results and they are available upon request.

[25] Throughout the quantitative analysis, we focus on an empirically realistic case of the inflation rate where $\pi \leq 15\%$. According to FRED, the maximum annual inflation rate for the US from 1960 to 2020 is 13.5%. Thus, we consider 0.15 as the upper bound of the inflation rate. Correspondingly, the upper bound of the nominal interest rate is 20.54% in the benchmark case based on the condition $i - g(i) - \rho = 15\%$.

Table 1: Calibration

Parameters Taken from External Sources			
Parameters	Interpretation		Value
ρ	Subjective discount rate		0.05
Calibrated Parameters and Targeted Moments			
Parameters	Value	Moments	Value
Quality step size of innovation, λ	1.1	Innovation arrival rate	6%
Intermediate-goods share, α	0.724	Labor share of GDP	58%
R&D productivity, φ	0.524	TFP growth rate	0.6%
CIA constraint on R&D, κ	0.398	Wealth-to-labor income ratio	6.45
Nominal interest rate, i	0.077	Inflation rate	2.1%

decreases from 0.64% to 0.54%. This negative nexus between the inflation rate and the economic growth rate is also in line with several theoretical findings such as Chu and Cozzi (2014) and Huang *et al.* (2017).

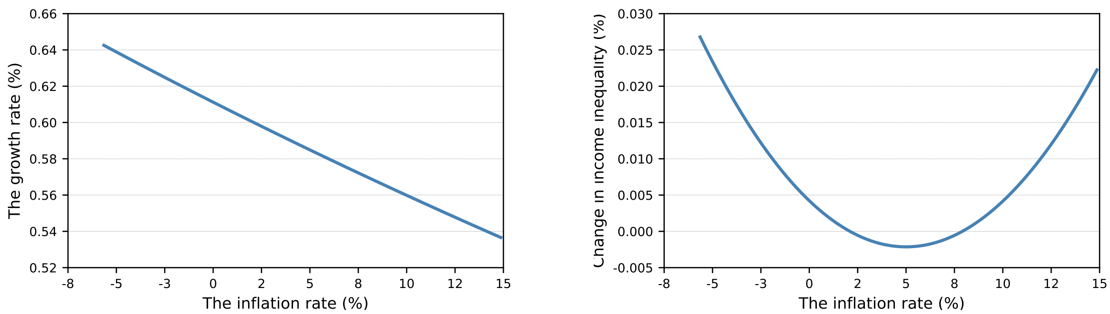

Figure 1: (a) Inflation and economic growth; (b) Inflation and income inequality.

Figure 1b simulates the relation between inflation and income inequality. Given the calibrated relative variance of wealth to skill distribution, the percentage in the coefficient of variation of income is found to be a U-shaped function of inflation rate. This finding is consistent with the theoretical prediction in Proposition 2 such that income inequality can be a U-shaped function of inflation rate (nominal interest rate) if $\Phi_{i=0} = 0.3511 < \sigma_h^2/\sigma_d^2 = 0.365$,[26] and this outcome is also consistent with the empirical finding in literature such as Bulíř (2001) and Balcilar *et al.* (2018), who use non-linear regressions with the US data as well as OECD data. Specifically, the coefficient of variation of income is minimized at an inflation rate of 5.0%. After that, any further increase in inflation rate is related to a rise in income inequality. For example, raising the

[26]Given our benchmark parameter values, for any $\alpha \in (0,1)$, the ratio of interest income to labor income Φ is monotonically increasing in the inflation rate according to Lemma 5. Therefore, the results in Proposition 2 always hold for any $\alpha > 0$.

inflation rate from 5.0% to 15% marginally enlarges income inequality by 0.03%. These results of inflation on economic growth and income inequality show that the government faces a trade-off between growth-maximizing and inequality-minimizing targets, although the benefits from reducing income inequality is fairly marginal.

4.3 Sensitivity analysis on intermediate-goods share of output

In this section, we examine the sensitivity of benchmark simulation results under alternative values of α. Bergholt *et al.* (2022) shows that although the economy-wide measure of labor's share of income in the United States fluctuated around $56-58\%$, the payroll measure of labor income share has fallen to around 54% after reaching its peak at the beginning of the new century. Considering the declining trend of the labor's share of income, we re-calibrate α, which governs the labor's share of income in our model, by setting $w_t h/(y_t - x_t) = 1/(1+\alpha) = 0.54$. We then obtain that $\alpha = 0.852$.[27] Remaining other parameter values unchanged as in the benchmark case, Figure 2a shows that the relation between inflation and economic growth in this case is still monotonically decreasing as in the benchmark case. However, the nexus between inflation and income inequality now follows a monotonically increasing pattern, as displayed in Figure 2b.[28] This outcome is also consistent with the theoretical prediction in Proposition 2 because the ratio of interest income to labor income in the zero-nominal-interest-rate equilibrium $\Phi_{i=0} = 0.748 > \sigma_h^2/\sigma_d^2 = 0.365$. More specifically, increasing the inflation rate from -5.13% (where the nominal interest rate is zero) to 15% (where the nominal interest rate is 20.08%) enlarges the degree of income inequality by 2.86%, and this rate of change is stable for change in inflation rate.

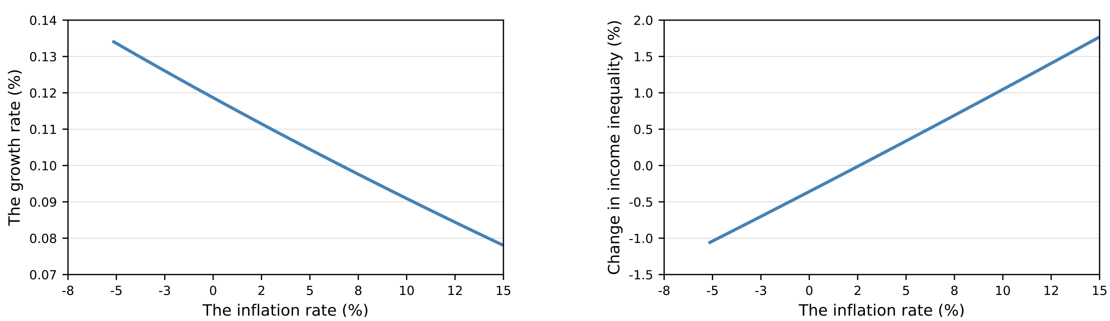

Figure 2: (a) Inflation and economic growth ($\alpha = 0.852$); (b) Inflation and income inequality ($\alpha = 0.852$)

[27]We have also considered a much lower yet empirically implausible value of α (e.g., 0.4) as it refers to a counter-factual and higher labor's share of income (i.e., 0.714). We find that the relation between inflation rate and income inequality is still U-shaped as in the benchmark case because $\Phi_{i=0} < \sigma_h^2/\sigma_d^2$ continues to hold. But the level of inequality-minimizing inflation rate becomes much larger in this case. The results are available upon request.

[28]We still take $i = 0.077$ in the benchmark as the initial equilibrium for simplicity.

4.4 The correlation (covariance) between wealth and skill heterogeneities

In this subsection, we relax our assumption on the independence between wealth and skill heterogeneities by considering their covariance, that is $\sigma_{d,h} \neq 0$. In this case, the variance of income distribution in (31) becomes $\sigma_I^2 = \left(\Phi^2\sigma_d^2 + 2\Phi\sigma_{d,h} + \sigma_h^2\right)/(1+\Phi)^2$. Differentiating it with respect to i yields

$$\frac{\partial \sigma_I^2}{\partial i} \gtrless 0 \Leftrightarrow \frac{\partial \Phi}{\partial i} \cdot \left[\Phi(\sigma_d^2 - \sigma_{d,h}) - (\sigma_h^2 - \sigma_{d,h})\right] \gtrless 0. \tag{37}$$

Notice that when $\sigma_{d,h} = 0$, this relation is reduced to (36).

Analogous to the discussion below equation (36), since $\partial\Phi/\partial i > 0$ holds for all i for a sufficiently large α, the relation between σ_I^2 and i depends on the sign of $\Phi(\sigma_d^2 - \sigma_{d,h}) - (\sigma_h^2 - \sigma_{d,h})$ in the LHS of (37). Suppose that the variance of wealth distribution is larger than the covariance between the wealth and skill distributions such that $\sigma_d^2 > \sigma_{d,h}$.[29] If the wealth heterogeneity dominates the skill heterogeneity in the zero-nominal-interest-rate equilibrium such that $\Phi_{i=0}(\sigma_d^2 - \sigma_{d,h}) > (\sigma_h^2 - \sigma_{d,h})$, then the variance of income distribution is monotonically increasing in the nominal interest rate because the fact Φ monotonically increases in i ensures $\Phi(\sigma_d^2 - \sigma_{d,h}) > (\sigma_h^2 - \sigma_{d,h})$ for all i in $[0, \hat{i}]$. Intuitively, a higher nominal interest rate enlarges the income inequality in that it always increases the relative contribution of wealth heterogeneity that features a relatively larger dispersion. Conversely, if the wealth heterogeneity is dominated by the skill heterogeneity in the zero-nominal-interest-rate environment such that $\Phi_{i=0}(\sigma_d^2 - \sigma_{d,h}) < (\sigma_h^2 - \sigma_{d,h})$, there may exist a threshold $i = \tilde{i} < \hat{i}$ below which the inequality $\Phi_{0<i<\tilde{i}}(\sigma_d^2 - \sigma_{d,h}) < (\sigma_h^2 - \sigma_{d,h})$ remains and above which the inequality is reversed to become $\Phi_{\tilde{i}<i<\hat{i}}(\sigma_d^2 - \sigma_{d,h}) > (\sigma_h^2 - \sigma_{d,h})$. In this circumstance, a higher nominal interest rate first reduces the income inequality and enlarges it afterwards, implying a U-shaped nexus between the income inequality and the nominal interest rate. However, if the threshold $i = \tilde{i} > \hat{i}$, then $\Phi_{0<i<\hat{i}}(\sigma_d^2 - \sigma_{d,h}) < (\sigma_h^2 - \sigma_{d,h})$ always holds, implying that the income inequality monotonically decreases in the nominal interest within its reasonable range of $[0, \hat{i}]$.

We next simulate the relation between inflation and income inequality.[30] In an empirical study, Pfeffer (2011) uses samples from NLSY and PSID data covering years around 2005 to 2007, which contain observations from the US families, and reports the correlation coefficient between family wealth and child education attainment, denoted as $\sigma_{d,h}$, to be in the range $[0.288, 0.376]$. We take the average value (i.e., 0.33) of a set of correlation coefficients documented in his study for $\sigma_{d,h}$.[31] Figure 3a shows that under a correlation coefficient $\sigma_{d,h} = 0.33$, the relation between

[29] This relation maintains given the calibrated parameter values. More importantly, this assumption also receives empirical support from Liu (2019). According to the moments reported by Liu (2019) who applies the Health and Retirement Study (HRS), a longitudinal study of Americans covering the period of 1992 to 2016, we compute the covariance between wealth and skill distributions and find that it is much less than the variance of wealth distribution.

[30] We do not report the results of inflation and economic growth in this experiment, because covariance does not enter the output/technology growth rate function.

[31] García-Peñalosa and Turnovsky (2015) also set the correlation between capital and skills endowments to 0.33 in their numerical analysis.

inflation and income inequality turns out to be positive. It is because the wealth heterogeneity continues to dominate the skill heterogeneity as the inflation rate/nominal interest rate rises such that $\Phi\sigma_d^2 + \sigma_{d,h} - \Phi\sigma_{d,h} - \sigma_h^2 > 0$ in (37), and the ratio of interest income to labor income is increasing in the inflation rate.

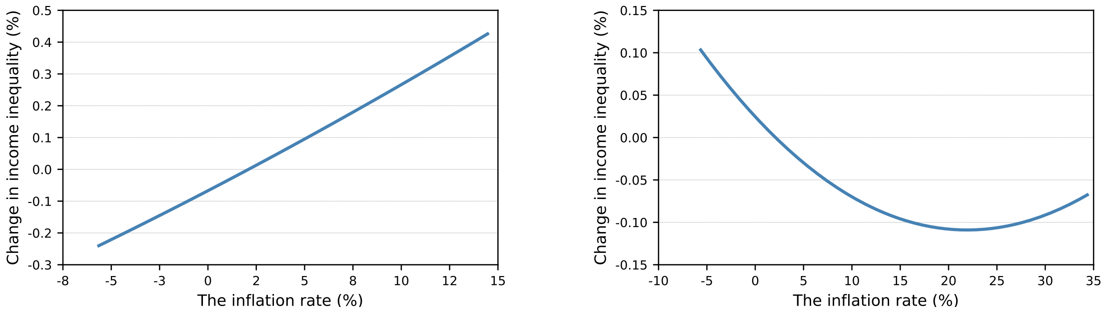

Figure 3: (a) Inflation and income inequality ($\sigma_{d,h} = 0.33$); (b) Inflation and income inequality ($\sigma_{d,h} = -0.1$)

We consider another case in which wealth distribution and skill distribution are negatively correlated. Following García-Peñalosa and Turnovsky (2012), we set the sign of correlation to -0.1. Figure 3b then indicates that the U-shaped relation between inflation and income inequality in the benchmark case re-emerges. In this case, the wealth heterogeneity is dominated by the skill heterogeneity in the zero-nominal-interest rate equilibrium such that $\Phi_{i=0}(\sigma_d^2 - \sigma_{d,h}) - (\sigma_h^2 - \sigma_{d,h}) < 0$ but the sign is reversed as the inflation rate continues to rise. Moreover, the level of inequality-minimizing inflation rate in this case becomes larger as compared to the benchmark case, reaching 21.92%.

5 Conclusion

In this study, we have developed a Schumpeterian growth model with two dimensions of heterogeneity among households. They are heterogeneous in terms of different levels of wealth and skill, which together generate an endogenous distribution of income. Within this monetary growth-theoretic framework, we find that inflation unambiguously leads to a decrease in the economic growth rate. However, the impact of inflation on income inequality depends on the relative variance of wealth distribution to skill distribution, and how inflation affects the ratio of interest income to labor income.

We also calibrate our model to the US data, and the benchmark simulation results tend to observe a U-shaped relation between inflation and income inequality. Finally, we perform several sensitivity analysis on some key structural parameters and consider the general case with a non-negative covariance between wealth and skill heterogeneities. Therefore, by allowing for both wealth and skill heterogeneities, our model is able to produce a mixed relation between inflation and income inequality, which provides a potentially novel mechanism that potentially reconciles the inconsistency in recent empirical findings.

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Appendix A

A.1 Derivations of the household optimality conditions.

The Hamiltonian equation is given by

$$H = e^{-\rho t} \ln c_t(s) + \omega_t(s)[r_t a_t(s) + i_t b_t(s) - \pi_t m_t(s) + w_t h(s) + \tau_t - c_t(s)] + \nu_t(s)\left[m_t(s) - b_t(s)\right], \tag{A.1}$$

where $\omega_t(s)$ and $\nu_t(s)$ are co-state variables. The first-order conditions for $c_t(s)$, $b_t(s)$, $a_t(s)$ and $m_t(s)$ are, respectively, given by

$$\frac{e^{-\rho t}}{c_t(s)} = \omega_t(s), \tag{A.2}$$

$$\omega_t(s) i_t = \nu_t(s), \tag{A.3}$$

$$\omega_t(s) r_t + \dot{\omega}_t(s) = 0, \tag{A.4}$$

$$-\pi_t\omega_t(s)+\nu_t(s)+\dot{\omega}_t(s)=0. \tag{A.5}$$

Substituting (A.3) into (A.5) to eliminate $\nu_t(s)$, and further making use of (A.4) yield the Fisher equation such that $i_t = r_t + \pi_t$. In addition, taking the log of (A.2) and differentiating the resulting equation with respect to t, and using (A.4) yield the Euler equation (4).

A.2 Proof of Lemma 1

In this proof, we examine the stability of this model given a stationary path of $[i_t]_{t=0}^{\infty}$. First, define the transformed variable $\Psi_t \equiv c_t/y_t$. Then, taking the log-differentiation of Ψ_t with respect to time yields

$$\frac{\dot{\Psi}_t}{\Psi_t}=\frac{\dot{c}_t}{c_t}-\frac{\dot{y}_t}{y_t}. \tag{A.6}$$

Using (15) and (20), we obtain

$$y_t=\frac{\varphi\lambda}{\alpha(1+\kappa i)}v_t, \tag{A.7}$$

Hence, (A.7) implies

$$\frac{\dot{y}_t}{y_t}=\frac{\dot{v}_t}{v_t}. \tag{A.8}$$

In addition, aggregating (12) for all ϵ yields

$$\begin{aligned}
&\int_0^1 r_t v_t(\epsilon|q)d\epsilon=\int_0^1\Pi_t(\epsilon|q)d\epsilon+\int_0^1\dot{v}_t(\epsilon|q)d\epsilon-\int_0^1\mu_t(\epsilon|q)v_t(\epsilon|q)d\epsilon\\
\Leftrightarrow & r_t v_t=\Pi_t+\dot{v}_t-\int_0^1\frac{\varphi z_t(\epsilon|q)}{hq_t(\epsilon)}\cdot\frac{hq_t(\epsilon)(1+\kappa i)}{\varphi\lambda}d\epsilon\\
\Leftrightarrow & r_t v_t=\Pi_t+\dot{v}_t-\int_0^1 z_t(\epsilon|q)\cdot\frac{1+\kappa i}{\lambda}d\epsilon\\
\Leftrightarrow & \frac{\dot{v}_t}{v_t}=r_t-\frac{\Pi_t}{v_t}+\frac{(1+\kappa i)z_t}{\lambda v_t}\\
\Leftrightarrow & \frac{\dot{v}_t}{v_t}=r_t-\frac{\Pi_t}{v_t}+\frac{(1+\kappa i)(y_t-c_t-x_t)}{\lambda v_t}.
\end{aligned} \tag{A.9}$$

The second equality applies the definitions of $v_t = \int_0^1 v_t(\epsilon|q)d\epsilon$, $\Pi_t = \int_0^1 \Pi_t(\epsilon|q)d\epsilon$, $\dot{v}_t = \int_0^1 \dot{v}_t(\epsilon|q)d\epsilon$ and $z_t = \int_0^1 z_t(\epsilon|q)d\epsilon$, and (13) and (14) in sequence. The last equality uses the final good market-clearing condition in (21). Inserting (15), (17) (19), and (20) into (A.9) yields

$$\begin{aligned}
\frac{\dot{v}_t}{v_t}&=r_t-\frac{(1-\alpha)Q_t h}{(1+\kappa i)Q_t h/\varphi\lambda}+\left(\frac{1+\kappa i}{\lambda}\right)\cdot\left\{\frac{Q_t h/\alpha}{(1+\kappa i)Q_t h/\varphi\lambda}-\frac{c_t}{y_t}\cdot\frac{Q_t h/\alpha}{(1+\kappa i)Q_t h/\varphi\lambda}-\frac{\alpha Q_t h}{(1+\kappa i)Q_t h/\varphi\lambda}\right\}\\
&=r_t-\frac{\varphi\lambda(1-\alpha)}{1+\kappa i}+\frac{\varphi}{\alpha}-\frac{\varphi}{\alpha}\Psi_t-\alpha\varphi.
\end{aligned} \tag{A.10}$$

Then, substituting (A.8) and (A.10) into (A.6), together with the Euler equation (4), yields a one-dimensional differential equation for Ψ_t such that

$$\begin{aligned}\frac{\dot{\Psi}_t}{\Psi_t} &= \frac{\dot{c}_t}{c_t} - \frac{\dot{y}_t}{y_t} = -\rho + \frac{\varphi\lambda(1-\alpha)}{1+\kappa i} - \frac{\varphi}{\alpha} + \frac{\varphi\Psi_t}{\alpha} + \alpha\varphi \\ &= \frac{\varphi\Psi_t}{\alpha} + \frac{\varphi\lambda(1-\alpha)}{1+\kappa i} - \rho - \frac{\varphi}{\alpha} + \alpha\varphi.\end{aligned} \tag{A.11}$$

Therefore, given that Ψ_t is a control variable and that its coefficient in (A.11) is positive, the dynamics of Ψ_t is characterized by saddle-point stability such that Ψ_t jumps immediately to its steady-state value given by

$$\Psi = 1 - \alpha^2 - \frac{\alpha\lambda(1-\alpha)}{1+\kappa i} + \frac{\alpha\rho}{\varphi}. \tag{A.12}$$

where the parameter space is restricted to ensure $\Psi > 0$. Given the stationarity of Ψ, (A.6) and (A.8) immediately follow that $\dot{c}_t/c_t = \dot{y}_t/y_t = \dot{v}_t/v_t$. In addition, (15) and (17) imply that $\dot{y}_t/y_t = \dot{x}_t/x_t$. Therefore, z_t should grow at the same rate with $\{y_t, c_t, x_t\}$ according to (21). From (23), we then have $\dot{b}_t/b_t = \dot{z}_t/z_t$. Moreover, in equilibrium the CIA constraint in (3) should be binding so that $\dot{b}_t/b_t = \dot{m}_t/m_t$. Finally, since the aggregate effective labor supply h is always stationary, we then have $\dot{y}_t/y_t = \dot{Q}_t/Q_t = \dot{w}_t/w_t$ according to (15) and (18). Eventually, we formally have

$$\frac{\dot{y}_t}{y_t} = \frac{\dot{v}_t}{v_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{Q}_t}{Q_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{x}_t}{x_t} = \frac{\dot{z}_t}{z_t} = \frac{\dot{b}_t}{b_t} = \frac{\dot{m}_t}{m_t}. \tag{A.13}$$

A.3 Proof of Lemma 2

First, for a given level of quality $q_t(\epsilon)$, which is constant between time t to $t + \Delta t$ until a new innovation comes into this line, the value of a firm in line ϵ (i.e., $v_t(\epsilon|q)$) should be constant, namely $\dot{v}_t(\epsilon|q) = 0$. Thus, from equation (12) we obtain

$$v(\epsilon|q) = \frac{\Pi(\epsilon|q)}{r + \mu(\epsilon|q)}, \tag{A.14}$$

where r, $\Pi(\epsilon|q)$ and $\mu(\epsilon|q)$ are the steady-state levels of real interest rate, monopolistic profit and arrival rate of successful innovation in line ϵ, respectively. Then, inserting (11) and (14) into (A.14) yields

$$r + \mu(\epsilon|q) = \frac{\varphi\lambda(1-\alpha)}{1+\kappa i}, \tag{A.15}$$

implying that along the BGP the arrival rate of the next successful innovation is the same for all product lines, denoted by μ. Accordingly, using (13), we can rewrite the aggregate R&D spending as

$$z_t = \int_0^1 \frac{\mu h}{\varphi} \cdot q_t(\epsilon) d\epsilon = \frac{\mu Q_t h}{\varphi}, \tag{A.16}$$

which is (24). By Lemma 1, substituting (A.15) into (4) yields

$$g = r - \rho = \frac{\varphi\lambda(1-\alpha)}{1+\kappa i} - \mu - \rho, \tag{A.17}$$

which is the steady-state growth rate of output and also that of technology.

We next derive the expression of the growth rate of aggregate quality index Q_t. By definition, in a time interval Δt, there are $\mu_t \Delta t$ sectors that experience one innovation that increases the productivity by λ. Therefore, the dynamics of Q_t is

$$Q_{t+\Delta t} = \mu_t \Delta t \int_0^1 \lambda q_t(\epsilon) d\epsilon + (1 - \mu_t \Delta t) \int_0^1 q_t(\epsilon) d\epsilon = Q_t[1 + \mu_t \Delta t \cdot (\lambda - 1)] \tag{A.18}$$

Now subtracting Q_t from both sides, dividing by Δt, and taking the limit as $\Delta t \to 0$ shows

$$g = \frac{\dot{Q}_t}{Q_t} = \mu(\lambda - 1), \tag{A.19}$$

where $\dot{Q}_t = \lim_{\Delta t \to 0}(Q_{t+\Delta t} - Q_t)/\Delta t$. Then, combining (A.17) and (A.19) yields the steady-state arrival rate of innovation in (25). Finally, substituting (25) into (A.19) yields the steady-state growth rate of aggregate quality in (26).

A.4 Proof of Lemma 3

Since (4) implies that $\dot{c}_t(s)/c_t(s) = \dot{c}_t/c_t = r_t - \rho$, we thus have

$$\frac{\dot{\theta}_{c,t}(s)}{\theta_{c,t}(s)} = \frac{\dot{c}_t(s)}{c_t(s)} - \frac{\dot{c}_t}{c_t} = 0.$$

Therefore, $\theta_{c,t}(s) = \theta_{c,0}(s)$ must hold for all time $t > 0$. Substituting this condition into (28) yields

$$\dot{\theta}_{d,t}(s) = \underbrace{\frac{c_t - w_t h - \tau_t}{d_t}}_{\chi_1} \theta_{d,t}(s) - \frac{c_t \theta_{c,0}(s) - w_t h \theta_{h,0}(s) - \tau_t}{d_t}. \tag{A.20}$$

According to Proposition 1, $\{c_t, a_t, b_t, m_t, d_t, \tau_t, w_t\}$ all grow at the same rate g on the balanced growth path.[32] Using the individual budget constraint in (4) and the aggregate budget constraint $\dot{d}_t = r_t d_t + w_t h - c_t + \tau_t$, we then have

$$\chi_1 = r_t - \dot{d}_t/d_t = \rho > 0. \tag{A.21}$$

[32]For an exogenously given i, the budget constraint for the monetary authority $\tau_t = (i - \rho)m_t$ implies $\dot{\tau}_t/\tau_t = \dot{m}_t/m_t = g$ according to Lemma 1.

Since $\theta_{d,t}(s)$ is a state variable and the coefficient of $\theta_{d,t}(s)$ is positive, the only solution for the one-dimensional differential equation that describes the potential evolution of $\theta_{d,t}(s)$ given an initial $\theta_{d,0}(s)$, as presented in (28), is $\dot{\theta}_{d,t}(s) = 0$ for all $t > 0$. This can be achieved by letting consumption share $\theta_{c,t}(s)$ jump to its steady state value $\theta_{c,0}(s)$. Imposing $\dot{\theta}_{dt}(s) = 0$ on (A.20) yields

$$\theta_{c,0}(s) = \frac{\rho d_t}{c_t} \cdot \theta_{d,0}(s) + \frac{w_t h}{c_t} \cdot \theta_{h,0}(s) + \frac{\tau_t}{c_t}. \tag{A.22}$$

A.5 Proof of Lemma 5

Differentiating Φ in (35) with respect to i yields

$$\begin{aligned}
&\frac{\partial \Phi}{\partial i} \gtrless 0 \\
\Leftrightarrow & \frac{-\kappa\varphi(1-\alpha)(\lambda-1)}{(1+\kappa i)^2}\left\{\frac{\alpha(1+\kappa i-\kappa\rho)}{\lambda\varphi(1-\alpha)} + \frac{\alpha\kappa}{1+\kappa i}\right\} + \left\{\frac{\varphi(1-\alpha)(\lambda-1)}{1+\kappa i} + \frac{\rho}{\lambda}\right\}\left\{\frac{\alpha\kappa}{\lambda\varphi(1-\alpha)} - \frac{\alpha\kappa^2}{(1+\kappa i)^2}\right\} \gtrless 0 \\
\Leftrightarrow & -\varphi(1-\alpha)(\lambda-1)\left\{\frac{(1+\kappa i)(1+\kappa i-\kappa\rho)}{\lambda\varphi(1-\alpha)} + \kappa\right\} + \left\{\varphi(1-\alpha)(\lambda-1) + \frac{\rho(1+\kappa i)}{\lambda}\right\}\left\{\frac{(1+\kappa i)^2}{\lambda\varphi(1-\alpha)} - \kappa\right\} \gtrless 0 \\
\Leftrightarrow & -\frac{(\lambda-1)(1+\kappa i)(1+\kappa i-\kappa\rho)}{\lambda} - \kappa\varphi(1-\alpha)(\lambda-1) + \frac{(\lambda-1)(1+\kappa i)^2}{\lambda} \\
& -\kappa\varphi(1-\alpha)(\lambda-1) + \frac{\rho(1+\kappa i)^3}{\varphi\lambda^2(1-\alpha)} - \frac{\kappa\rho(1+\kappa i)}{\lambda} \gtrless 0 \\
\Leftrightarrow & \frac{\kappa\rho(\lambda-1)(1+\kappa i)}{\lambda} - 2\kappa\varphi(1-\alpha)(\lambda-1) + \frac{\rho(1+\kappa i)^3}{\varphi\lambda^2(1-\alpha)} - \frac{\kappa\rho(1+\kappa i)}{\lambda} \gtrless 0 \\
\Leftrightarrow & \frac{\kappa\rho(\lambda-2)(1+\kappa i)}{\lambda} - 2\kappa\varphi(1-\alpha)(\lambda-1) + \frac{\rho(1+\kappa i)^3}{\varphi\lambda^2(1-\alpha)} \gtrless 0 \\
\Leftrightarrow & \frac{\kappa\rho(\lambda-2)}{\lambda} + \frac{\rho(1+\kappa i)^2}{\varphi\lambda^2(1-\alpha)} - \frac{2\kappa\varphi(1-\alpha)(\lambda-1)}{1+\kappa i} \gtrless 0 \\
\Leftrightarrow & \frac{\rho}{\varphi\lambda^2(1-\alpha)}\left\{-\kappa\varphi\lambda(2-\lambda)(1-\alpha) + (1+\kappa i)^2\right\} - \frac{2\kappa\varphi(1-\alpha)(\lambda-1)}{1+\kappa i} \gtrless 0
\end{aligned} \tag{A.23}$$

where the second inequality is obtained by multiplying the first inequality by $(1+\kappa i)^3/\alpha k$ and the second last inequality is derived by dividing the third last inequality by $(1+\kappa i)$.

To examine $\partial\Phi/\partial i \gtrless 0$, let us further denote the left-hand-side of (A.23) by

$$\Theta = \frac{\rho}{\varphi\lambda^2(1-\alpha)}\left\{-\kappa\varphi\lambda(2-\lambda)(1-\alpha) + (1+\kappa i)^2\right\} - \frac{2\kappa\varphi(1-\alpha)(\lambda-1)}{1+\kappa i}. \tag{A.24}$$

To understand the relationship between Φ and i, we need to examine $\partial\Theta/\partial i$. In examining $\partial\Theta/\partial i$, we restrict our attention to the feasible range of i only, i.e, the lower bound is 0, and the upper bound, denoted as $\hat{i}$, that ensures a non-negative innovation arrival rate μ as in equation (25).

That is, $\mu(i) = 0$ decides $\hat{i}$ such that

$$\hat{i} \equiv (\lambda\varphi(1-\alpha)/\rho - 1)/\kappa. \tag{A.25}$$

μ is non-negative for any level below $\hat{i}$.

The proof consists of several steps.

Step 1: We show that

$$\frac{\partial\Theta}{\partial i} = \frac{2(1-\alpha)\kappa^2(\lambda-1)\phi}{(i\kappa+1)^2} + \frac{2\kappa\rho(i\kappa+1)}{(1-\alpha)\lambda^2\phi} > 0,$$

meaning that Θ is monotonically increasing in i. Therefore, Θ reaches maximum when i reaches its upper bound $\hat{i}$, denoted as $\Theta_{i=\hat{i}}$, and minimum when i reaches its lower bound 0, denoted as $\Theta_{i=0}$.

Step 2: We investigate the sign of $\Theta_{i=\hat{i}}$ and will show that $\Theta_{i=\hat{i}} > 0$ for $\rho < \min[1/\lambda, \lambda\varphi(1-\alpha)]$.

First, substituting (A.25) into (A.24) gives the expression for the maximum of Θ:

$$\begin{aligned}
&\Theta_{i=\hat{i}} = \frac{\rho}{\varphi\lambda^2(1-\alpha)}\left\{\kappa\varphi\lambda(\lambda-2)(1-\alpha) + \frac{\lambda^2\varphi^2(1-\alpha)^2}{\rho^2}\right\} - \frac{2\rho\kappa\varphi(1-\alpha)(\lambda-1)}{\lambda\varphi(1-\alpha)} \gtrless 0\\
&\Leftrightarrow \frac{\rho}{\lambda}\left\{\kappa(\lambda-2) + \frac{\lambda\varphi(1-\alpha)}{\rho^2}\right\} - \frac{2\rho\kappa(\lambda-1)}{\lambda} \gtrless 0\\
&\Leftrightarrow \frac{\rho\kappa(\lambda-2-2\lambda+2)}{\lambda} + \frac{\varphi(1-\alpha)}{\rho} \gtrless 0\\
&\Leftrightarrow \varphi(1-\alpha) \gtrless \rho^2\kappa.
\end{aligned} \tag{A.26}$$

Next, we will verify that $\varphi(1-\alpha) > \rho^2\kappa$ holds under certain parameter conditions. Note that from $\mu \geq 0$ as in (25), it must hold that $\varphi(1-\alpha)/(1+\kappa i) \geq \rho/\lambda$ for all $i \geq 0$, which gives rise to the following inequality:

$$\varphi(1-\alpha) > \frac{\varphi(1-\alpha)}{(1+k\hat{i})} > \frac{\rho}{\lambda} \tag{A.27}$$

given $1 + \kappa\hat{i} > 1$. We further impose a reasonable restriction:

$$\rho < 1/\lambda, \tag{A.28}$$

which holds for a wide range of ρ and λ. Given (A.27), (A.28) and $0 < \kappa < 1$, it follows that $\varphi(1-\alpha) > \varphi(1-\alpha)/(1+\kappa\hat{i}) > \rho/\lambda > \rho^2 > \rho^2\kappa$, and therefore $\Theta_{i=\hat{i}} > 0$, from (A.26). We combine (A.27) and (A.28) to yield the condition for $\Theta_{i=\hat{i}} > 0$ to hold: $\rho < \min[1/\lambda, \lambda\varphi(1-\alpha)]$.

Step 3: In this step, we will examine the sign of $\Theta_{i=0}$, and show that there is a threshold value α_T above which $\Theta_{i=0}$ can be positive for $\rho < \min[1/\lambda, \lambda\varphi(1-\alpha)]$ and below which $\Theta_{i=0}$ can be negative for $\rho < \min[1/\lambda, \lambda\varphi(1-\alpha), \varphi\lambda\kappa\rho(2-\lambda)(1-\alpha) - 2\varphi\lambda(\lambda-1)(1-\alpha)^2]$.

By plugging $i = 0$ into (A.24) we obtain

$$\Theta_{i=0} = \frac{\rho[-\kappa\varphi\lambda(2-\lambda)(1-\alpha)+1]}{\varphi\lambda^2(1-\alpha)} - 2\kappa\varphi(1-\alpha)(\lambda-1) \gtrless 0$$
$$\Leftrightarrow \rho - \kappa\rho\varphi\lambda(2-\lambda)(1-\alpha) - 2\kappa\varphi^2\lambda^2(1-\alpha)^2(\lambda-1) \gtrless 0. \tag{A.29}$$

We then show that $\Theta_{i=0} > 0$ if α is sufficiently large. Recall that (A.27) implies $\alpha < 1 - \rho/\lambda\varphi$. Therefore, when $\alpha \to 1 - \rho/\lambda\varphi$,

$$\lim_{\alpha\to(1-\rho/\lambda\varphi)} \Theta_{i=0} = \rho - \frac{\kappa\rho^2\varphi\lambda(2-\lambda)}{\lambda\varphi} - \frac{2\kappa\rho^2\varphi^2\lambda^2(\lambda-1)}{(\lambda\varphi)^2} = \rho(1-\kappa\lambda\rho) > 0 \tag{A.30}$$

given that $1 > \rho\lambda > \rho\lambda k$. Therefore, for sufficiently large α, we have $\Theta_{i=0} > 0$.

On the other hand, if $\alpha \to 0$, we obtain

$$\lim_{\alpha\to 0} \Theta_{i=0} = \rho - \kappa\rho\varphi\lambda(2-\lambda) - 2\kappa\varphi^2\lambda^2(\lambda-1) \gtrless 0$$
$$\Leftrightarrow \frac{\rho}{\varphi\lambda} - \kappa\left[\rho(2-\lambda) - 2\varphi\lambda(\lambda-1)\right] \gtrless 0. \tag{A.31}$$

Obviously, the negative sign holds if $\rho/(\varphi\lambda) < \kappa\left[\rho(2-\lambda)(1-\alpha) - 2\varphi\lambda(\lambda-1)(1-\alpha)^2\right]$ as in (A.29). We impose this condition so that the sign in (A.31) is positive. Recall that $\Theta_{i=0}$ is monotonically increasing in α, we can ensure that there exists a unique threshold value α_T that solves $\Theta_{i=0}(\alpha_T) = 0$, above which $\Theta_{i=0} > 0$ and below which $\Theta_{i=0} < 0$ by the intermediate value theorem. Combining the above parameter restrictions yields the condition for $\rho : \rho < \min(1/\lambda, \lambda\varphi(1-\alpha), \varphi\lambda\kappa[\rho(2-\lambda)(1-\alpha) - 2\varphi\lambda(\lambda-1)(1-\alpha)^2])$.

Summing up the above proof, we eventually obtain the following conclusion:

(a) If $\alpha \in [\alpha_T, 1-\rho/\lambda\varphi)$ (i.e., α is sufficiently large), both $\Theta_{i=0} > 0$ and $\Theta_{i=\hat{i}} > 0$ hold, given the monotonicity of Θ in i, it follows that Φ is a monotonically increasing function of i for $\rho < \min[1/\lambda, \lambda\varphi(1-\alpha)]$.

(b) If $\alpha \in [0, \alpha_T)$ (i.e., α is sufficiently small), we have $\Theta_{i=0} < 0$ and $\Theta_{i=\hat{i}} > 0$. We therefore obtain that Φ is a U-shaped function of i for $\rho < \min(1/\lambda, \lambda\varphi(1-\alpha), \varphi\lambda\kappa[\rho(2-\lambda)(1-\alpha) - 2\varphi\lambda(\lambda-1)(1-\alpha)^2])$.

Appendix B

In this appendix, we relax the assumption of $\eta = \alpha$ and show that the analytical results of the model still hold. For $\eta \neq \alpha$, the optimal monopolistic price in equation (9) becomes

$$p_t(\epsilon|q) = \frac{\eta q_t(\epsilon)}{\alpha}. \tag{B.1}$$

The quantity of intermediate goods in industry ϵ is

$$x_t(\epsilon|q) = \left(\frac{\alpha}{\eta}\right)^{\frac{1}{1-\alpha}} h, \tag{B.2}$$

and the monopolistic flow profits of a firm in (11) turns to be

$$\Pi_t(\epsilon|q) = (1-\alpha)q_t(\epsilon)\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}} h. \tag{B.3}$$

By substituting (B.2) into (5), we derive the total output as

$$y_t = \frac{Q_t h}{\alpha}\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}} \tag{B.4}$$

The aggregate expenditure on final good used to produce intermediate goods is now

$$x_t = \eta\left(\frac{\alpha}{\eta}\right)^{\frac{1}{1-\alpha}} Q_t h, \tag{B.5}$$

which replaces (17). Substituting (B.4) into (7) yields the equilibrium real wage rate

$$w_t = \left(\frac{1-\alpha}{\alpha}\right) Q_t \left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}}, \tag{B.6}$$

instead of (18). The total profits of the intermediate-goods sector become

$$\Pi_t = (1-\alpha)\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}} Q_t h. \tag{B.7}$$

Moreover, the market aggregate value of firms in the intermediate goods sector remains unchanged as in the case with $\eta = \alpha$.

B.1 Effects of monetary policy on innovation and growth

Inserting (B.3) and (14) into (A.14) yields

$$r + \mu(\epsilon|q) = \frac{(1-\alpha)q_t(\epsilon)\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}} h}{q_t(\epsilon)h(1+\kappa i)/\varphi\lambda} = \frac{\varphi\lambda(1-\alpha)}{1+\kappa i}\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}}, \tag{B.8}$$

which replaces (A.15). Then substituting (B.8) into (4) shows

$$g = r - \rho = \frac{\varphi\lambda(1-\alpha)}{1+\kappa i}\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}} - \mu - \rho, \tag{B.9}$$

instead of (A.17). Combing (B.9) and (A.19), we derive the steady-state arrival rate of innovation and the growth rate of aggregate quality such that

$$\mu = \frac{\varphi(1-\alpha)}{1+\kappa i}\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}} - \frac{\rho}{\lambda}, \tag{B.10}$$

and

$$g = \frac{\varphi(1-\alpha)}{1+\kappa i}\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}}(\lambda-1) - \frac{\rho(\lambda-1)}{\lambda}, \tag{B.11}$$

which replace (25) and (26), respectively. We can see that the assumption of $\eta \neq \alpha$ does not qualitatively affect the effects of monetary policy on innovation and economic growth.

B.2 Effects of monetary policy on income inequality

The expression of financial assets in (33) remains unchanged. The amount of borrowing b_t in (34) becomes

$$b_t = \frac{\kappa Q_t h}{\varphi}\left[\frac{\varphi(1-\alpha)}{1+\kappa i}\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}} - \frac{\rho}{\lambda}\right]. \tag{B.12}$$

Combining (33) and (B.12), along with (B.6), we derive the ratio of wealth to labor income given by

$$\begin{aligned} \frac{d_t}{w_t h} &= \frac{Q_t h(1+\kappa i)}{\varphi\lambda\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}} Q_t h} + \frac{\frac{\kappa Q_t h}{\varphi}\left[\frac{\varphi(1-\alpha)}{1+\kappa i}\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}} - \frac{\rho}{\lambda}\right]}{\left(\frac{1-\alpha}{\alpha}\right) Q_t h\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}}} \\ &= \frac{(1+\kappa i)}{\varphi\lambda\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}}} + \frac{\kappa\left[\frac{\varphi(1-\alpha)}{1+\kappa i}\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}} - \frac{\rho}{\lambda}\right]}{\varphi\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}}} \\ &= \frac{\alpha(1+\kappa i-\kappa\rho)}{\varphi\lambda(1-\alpha)\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}}} + \frac{\alpha\kappa}{1+\kappa i}, \end{aligned} \tag{B.13}$$

instead of (35). Then, by using (B.11), (B.13) and (4), we derive the ratio of interest income to labor income such that

$$\Phi = \left\{\frac{\varphi(1-\alpha)(\lambda-1)}{1+\kappa i}\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}} + \frac{\rho}{\lambda}\right\}\cdot\left\{\frac{\alpha(1+\kappa i-\kappa\rho)}{\varphi\lambda(1-\alpha)\left(\frac{\alpha}{\eta}\right)^{\frac{\alpha}{1-\alpha}}} + \frac{\alpha\kappa}{1+\kappa i}\right\}, \tag{B.14}$$

which replaces (36). We can see that the consideration of $\eta \neq \alpha$ only creates an exogenous term $(\alpha/\eta)^{\frac{\alpha}{1-\alpha}}$, which however does not qualitatively affect the effects of monetary policy on Φ and therefore σ_I^2.